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Unified model studies of N=84 and N=80 nuclei

Thomas Michael Corrigan
Iowa State University

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Thomas Michael Corrigan

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CHAPTER 1. INTRODUCTION

Since the discovery of the atomic nucleus by Rutherford, and the later discovery of the neutron by Chadwick (1), the task of the nuclear theorist has been to understand measured nuclear properties in terms of the interaction between the constituents of nuclei, protons and neutrons. Among the quantities which must be accounted for are the energy, spin, and parity of the nuclear states (both the unexcited (ground) and excited states) together with quantities which measure how the nucleus interacts electromagnetically. The latter involves multipole moments (2) of the nuclear states and transition rates between these states. A basic question arises: which properties of the nucleus can be explained in terms of one or two nucleons acting alone, and which properties are due to many nucleons acting together (correlations) to produce collective effects. An example of this distinction on an everyday scale would be a spinning, phosphorescent top. The individual atoms in the top are responsible for the phosphorescence by emitting light (photons) as atomic electrons de-excite to lower energy levels. The motion of these individual atoms is not independent, however; it is highly "correlated" by the electromagnetic forces acting between atoms and molecules to bind them together. The first suggestion that nucleons could undergo such collective motion was made as early as 1930 by Thibaud (3), but nuclear experimental information was too meagre to either confirm or deny such speculation at the time. In 1938 Teller and Wheeler (4) proposed that the nucleus might exhibit energy levels (spectra) corresponding to that

of a quantized rotor, in analogy with similar behavior already seen in molecular spectra (5), but this behavior was not seen until 1954 (6).

Much of the early success in describing nuclear properties was in terms of the actions of individual nucleons. Schmidt (7) was able to predict rather crudely the magnetic moments of nuclei in their ground state by assuming that each pair of nucleons couples its angular momentum to zero, and hence the angular momentum of the nucleus was simply that of the last (unpaired) nucleon. This approach led to the well-known shell-model (8-12), which postulates that each nucleon moves in a potential well-produced by the average interaction of all the nucleons. This model has been very successful in predicting ground state spins and parities, alpha and beta decay systematics, and the famous "magic" numbers (N or Z = 8, 14, 20, 28, 40, 50, 82, 126). Nuclei with these values of N or Z exhibit especially stable configurations.

The first serious consideration that some nuclear properties were collective in nature was made in connection with fission (13-14), where the shape and stability of a deformed liquid drop were studied. The total energy is considered to be a function of the deformation from a spherical equilibrium shape. Later, in connection with nuclear structure it became evident that many quadrupole moments were much too large to be accounted for by single particle shell-model estimates (12). The same is true for electric quadrupole transition rates. (See Figure 1) This led Rainwater (15) to suggest that a single odd particle could polarize the core, and thus a single particle could have a lower potential energy if its potential well were deformed. The deformed core then

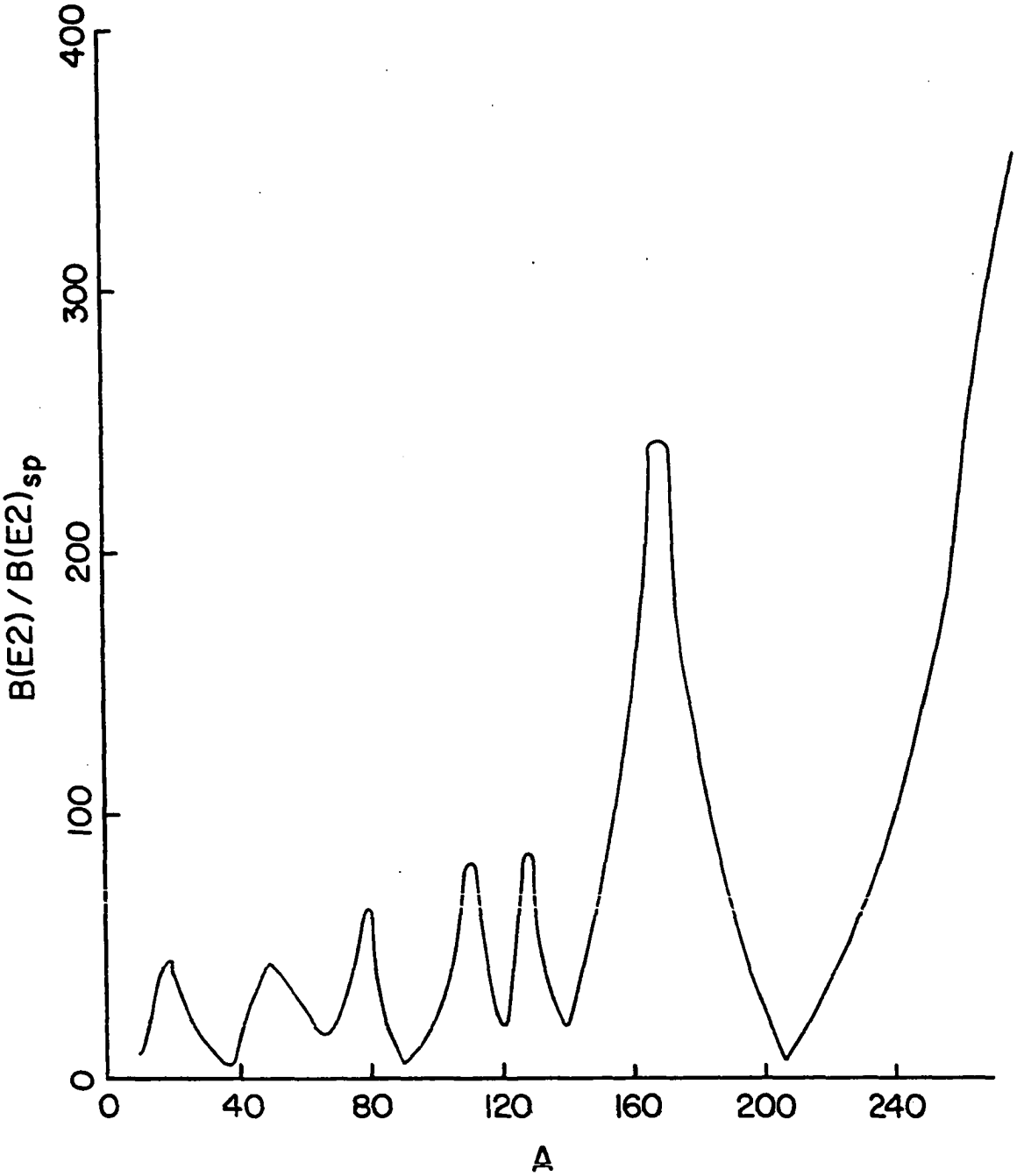

Figure 1. Experimental quadrupole transition probabilities. The ratio $B(E2; 0_1^+ \rightarrow 2_1^+)/B(E2)_{sp}$ is plotted with the single particle value given as $B_{sp}(E2) = \frac{5}{4} e^2 (\frac{3}{5} R^2)^2$. The curve is meant to show the trends as A increases; the data actually shows considerable scatter about the curve particularly for the lower A values.

contributes to the quadrupole moment. It was soon suggested that enhanced E2 transitions implied collective effects associated with deformation of the nuclear shape (16-17). One then realized that there were two especially simple modes of collective excitation which could be applied to a discussion of the nucleus, rotation and vibration. Near magic numbers (closed shells) nuclei are apparently spherical and the low lying excitations are expected to be vibrations about this spherical equilibrium shape. Far from closed shells nuclei are thought to be more strongly deformed by the valence nucleons, and indeed these nuclei exhibit rotational characteristics. It is the vibrational models which will concern us in this thesis.

Elementary excitation modes in a quantum system were first suggested by Landau (18) in connection with the excitation spectrum of superfluid liquid helium. Examples of collective vibrations in quantum systems are well-known from the study of molecules. There the atoms form a rigid structure, and low energy internal excitations correspond to normal vibrations of that structure (19).

Bohr and Mottelson (20, 21) recognized that the static shape and orientation of a deformed nucleus and the collective deformation variables of a spherical nucleus were related to each other; the shape and deformation variables play the role of dynamic coordinates. In 1952 Bohr (20) published the results of his study of the quantum theory of surface oscillations, in which he derived the basic form for the solutions to a quadrupole vibrator, and gave the symmetries of the wavefunctions. He was also the first to consider the problem which we are

concerned with here, the coupling of single particle motion to the nuclear surface oscillations. Bohr considered the weak coupling limit in which the effect of the core-particle coupling may be treated as a perturbation, and also the strong coupling limit in which the particle motion is considered relative to the symmetry axis of the nucleus. In the latter case, j_3, the projection of the particle's angular momentum onto the z (body) axis of the nucleus, and I_3, the projection of the total angular momentum on the z axis are constants of the motion. The description of the nucleus in terms of a core plus valence nucleons has come to be known as the unified model of nuclear dynamics.

If the coupling is neither very weak nor very strong the Hamiltonian must be diagonalized in the coupled basis of core states plus single particle states. This problem was first investigated by Bohr and Mottelson (22). The behavior of the energy levels and quadrupole moments as a function of the strength of the coupling interaction was studied by Choudhury (23) for the case of a single valence nucleon, and by Ford and Levinson (24) for the case of one and two nucleons in the $(f_{7/2})^2$ shell with weak coupling. No nuclei were actually investigated in these papers, but the behavior of the model was studied, and it was seen that the existing experimental evidence indicated that the vibrating core model should be applicable near closed shells.

The first investigation which considered the residual interaction of two valence nucleons was that of Raz (25), who treated the even-even nuclei as a whole, and found that the energy levels and BE2 ratios for $2_2^+ \rightarrow 2_1^+/2_1^+ \rightarrow 0_1^+$ could be accounted for better in this model than in

the pure vibrational model. The notation here means the ratio of the second 2^+ level to first 2^+ level E2 transition to that from the first 2^+ level to the 0^+ ground state. The form of the interaction used was a Bartlett (26) potential with Gaussian radial dependence.

Progress in applying the intermediate coupling unified model was slowed by the fact that this model required diagonalization of large matrices, so until modern computers began to come into general use not much was done. Here, we will briefly discuss some applications of this model in various situations, leading up to the work of Vanden Berghe which stimulated this work. This is not meant to be an exhaustive treatment, and references to some early work (prior to 1968) can be found in the article by Alga (27). We discuss mostly one and two valence particles (or holes), although some work has been done with three (28) and even four (29, 30) valence nucleons; these treat very small model spaces for the particles.

The odd-mass Promethium isotopes were investigated by Choudhury and O'Dwyer (31), by assuming that the core is the neighboring doubly-even Nd nucleus and then coupling the odd proton to this core. The proton was allowed to occupy the $g_{7/2}$ and $d_{5/2}$ states and up to 3 phonon excitations of the core were considered. The phonon energy, $\hbar\omega$, was taken to be fixed at the energy of the first 2^+ state in Nd, while ξ, the coupling constant and the $d_{5/2} - g_{7/2}$ energy difference $E_{5/2} - E_{7/2}$ were varied. The general features of the spectra were reproduced for the lowest states, but the ordering was not very good. Likewise the electromagnetic properties were the right order of magnitude. Some questionable

simplifications were made such as not treating the $g_{7/2}$ state as a hole and allowing the valence proton to be in any of the $d_{5/2}$ states when they are actually partly filled.

Heyde and Brussaard (32) looked at the odd proton nuclides $^{121, 125}Sb$, $^{129, 131}I$, ^{143}Pr, and $^{147, 149}Pm$, along with doubly even N = 84 nuclei ^{142}Ce, ^{144}Nd, and ^{146}Sm. In the case of the 2 valence neutrons they were forced to so severely restrict the orbitals available that only the $f_{7/2}$ orbit could be occupied. They used the same spin exchange residual interaction as Raz (25) and up to three phonons of core excitation. The low lying spectra were fairly well-reproduced, partly due to the scarcity of experimental levels at the time (1967). The magnetic dipole moment of the first excited state in ^{144}Nd was predicted to be smaller than actually found unless an unrealistically small value of the spin gyromagnetic ratio, g_s, was assumed. The theoretical quadrupole moments and reduced E2 transitions were in poor agreement with experiment, even using an effective neutron charge of $e_n^{eff} = e$. The quadrupole moment of the 2_1 state was even predicted to have the wrong sign. The solutions to these problems were all improved later by treating more orbitals and this thesis is, in part, an attempt to introduce a more realistic means for treating basis truncation.

The nuclei with one valence neutron hole in an N = 82 core were treated, first for ^{139}Ce (33), and then for ^{133}Te, ^{135}Xe, ^{137}Ba, ^{139}Ce, ^{141}Nd, and ^{143}Sm (34). In these calculations the neutron allowed to occupy any of the hole orbitals $3s_{1/2}$, $2d_{3/2}$, $2d_{5/2}$, $1g_{7/2}$, $1h_{11/2}$. The spacing of these single particle levels, $\hbar\omega$ and ξ, were then determined

by best agreement with experimental data. The energy spectra and gamma branching ratios were well-reproduced. The same procedure applied to the N = 83 nuclei (35) gave particle level spacings for the $3p_{1/2}$, $3p_{3/2}$, $2f_{5/2}$, $2f_{7/2}$, $1h_{9/2}$, and $1i_{13/2}$ orbitals, and similar good agreement to spectra, spectroscopic factors, and certain (not all) electromagnetic properties was obtained.

One attempt to keep the radial dependence in the core particle coupling was made by Vanden Berghe and Heyde (36) in a calculation for the odd-A Antimony isotopes. They used a Woods-Saxon type potential, and took the well parameters from optical model fits to (^{3}He, d) data (37). They found excellent agreement with the experimental levels but only fair agreement to the branching ratios. It was more trouble than it was worth, and neither they nor anyone else has worried about the radial dependence since then.

Some other regions where the basic core plus one valence nucleon model has been used successfully are the odd-mass Xenon isotopes (38), the odd-A isotopes of Sb, I, Cs, and Pr (39), and I^{115} (40). Since the calculations in this thesis are for 2 valence nucleons, we will not consider these in any further detail.

The even Tellurium isotopes were studied by Lopac (41) and Degrieck and Vanden Berghe (42). In the calculation of Lopac, the 2 valence protons were allowed to occupy the $3s_{1/2}$, $2d_{3/2}$, $2d_{5/2}$, $1g_{7/2}$, and $1h_{11/2}$ orbitals, while the core has allowed up to three phonon excitation and high lying two particle states were thrown away in an unspecified manner. The residual interaction was taken to be a simple pairing force affecting

only states with J = 0. The lower energy levels were well produced in the calculation by Lopac and the sign of the quadrupole moment of the first 2^+ state in ^{124}Te was correctly predicted, whereas an earlier calculation (43) using only the $g_{7/2}$, $d_{3/2}$, and $h_{11/2}$ had got it wrong. Degrieck and Vanden Berghe used a surface delta interaction for the residual interaction and looked at more Te isotopes since more experimental data had become available. The sign and magnitude of the quadrupole moments of the first 2^+ state were well-reproduced, but the actual numbers were not very close. In a separate paper (44), these same authors also considered the isomeric 6^+ state in the Te isotopes and showed that the lifetime of this state is explained quite well by this model. The quadrupole transitions between the 2_1^+ and 0_1^+ were also found to be in decent accord with experiment.

In a similar calculation (45) Alga, Paar, and Lopac correctly calculated the sign and magnitude of the quadrupole moments in even-A Fe, Zn, Cd, Te, and Hg isotopes, using a simple pairing residual interaction.

The N = 84 nuclei were treated by Vanden Berghe (46). He used a pairing residual interaction between the two neutrons outside the core and treated core states with up to three phonons. The single particle energies were taken from the calculation for the N = 83 nuclei (35) which were referred to earlier. The parameters $\hbar\omega$, ξ, and G (the strength of the pairing interaction) were varied until good overall agreement was obtained for the mass region being considered. Because the configuration space was too large to handle, Vanden Berghe truncated the basis by only considered two particle clusters with an unperturbed

energy up to 2.8 MeV. The general energy level spacing and ordering were reproduced, and inhibition of the crossover transition from the 2_2 state to the ground state 0_1, relative to the 2_2 to 2_1 transition was correctly predicted. In addition, the right sign for the quadrupole moment of the 2_1 state was found; the incorrect sign had been a defect of a previous calculation (32) by Heyde and Brussaard. Good agreement was also found with the experimental values that were known for magnetic dipole moments, electric quadrupole moments, and B(E2)'s. The eigenvectors found by Vanden Berghe were used to analyze the experimental data for ^{140}Ba found by the TRISTAN facility at Iowa State (47). Branching ratios and multipolarity mixing ratios were calculated and compared to experiment. Reasonable qualitative agreement with observed experimental facts was found, but some indication that higher phonon states might be needed was clearly indicated. It seemed to us that both the spectra and electromagnetic properties could be better reproduced if more phonons were treated and a more realistic basis truncation procedure was used. Since we had just solved the eigenfunction problem for the Bohr Hamiltonian in a multiplicity resolved form, it was feasible for the first time to undertake this calculation if a suitable procedure could be found to truncate the basis space to a manageable size. The solution to this problem, along with the first closed form expression for matrix elements of the collective surface parameter α, and the application of these ideas to some nuclei, are what is presented in the remainder of this thesis.

First we will note a few variations on the "classical" unified model which we have been discussing. There have been a small number of

attempts to treat the negative parity levels which are found in these nuclei. Paar (48) treated ^{63}Cu with a core and one valence proton, plus allowing particle-hole excitation of the core. In a similar approach Heyde, Waroquier, and Vincx (49) treated N = 83 nuclei by coupling neutron hole states in the N = 82 core to low lying levels of the odd neutron N = 83 nuclei to produce negative parity states. In the second part of the Vanden Berghe paper which we discussed earlier (46), the negative parity states were treated by considering octupole vibrations of the core and by treating the core-particle coupling as a perturbation.

In a study of ^{58}Ni (50), Vanden Berghe introduced the idea of "natural values" for the parameters. The single particle energies were taken from experiment and the first excited state of the ^{56}Ni core was taken as $\hbar\omega$, the phonon spacing. The pairing strength was assumed to be G = 27/A MeV and the coupling strength was derived from the experimental $B(E2; 2_1 \rightarrow 0_1)$ for the core. Although Vanden Berghe obtained good agreement with the energy spectra and most of the electromagnetic properties, he was forced to introduce not only an effective neutron charge but also an effective core charge. This seems inconsistent. If the rationale for introducing e_n^{eff} is to account phenomenologically for core polarization one should not have to do it again for the core. Vanden Berghe also took $e_n^{eff} = e$, which is quite large. More fundamentally, one would expect the two valence nucleons to have an effect on the core spacing. Even with all these modifications the mixing ratios were not really very well predicted.

The fact that the unified model assumes a vibrational form for the core did not seem realistic to Dang et al. (51). They attempt to derive phenomenological models from the shell-model and have some success. An attempt to treat the anharmonic effects of the core was presented in a paper by Castel, Stewart, and Harvey (52) who added a term

$$\sum_{\mu} \frac{1}{4} \sum_{J=0,2,4} \omega\eta_J (b_\mu^+ \; b_\mu^+)^J (b_\mu \; b_\mu)^J$$

to the quadrupole vibrator Hamiltonian and looked at nuclei in the 2s-1d shell with one valence proton. The effects of anharmonic terms on the quadrupole vibrator have also been studied in great detail by Gneuss et al. (53-55), and more recently by F. Margetan (56). These are not unified model calculations but only consider even-even nuclei as anharmonic nuclei. Lipas et al. (57-59) has gone a slightly different route; he treats harmonic vibrations about an axially deformed and more generally an arbitrary anisotropic shape. (This is as opposed to the normal treatment of harmonic vibrations about a spherical equilibrium.) In both the Gneuss and the Lipas models odd-A nuclei could be treated in principle by coupling the odd-A nucleon to the core. Such an undertaking would be both interesting and very difficult.

The remainder of this thesis then is the results obtained in our investigation of N = 84 and N = 80 nuclei using the unified model of Bohr and Mottelson (21, 22). The first part of the work is contained in an already published paper (60) which presents the multiplicity resolved wavefunctions for quadrupole vibrations about a spherical equilibrium

shape. In Chapter 2, the mathematics of the model are worked out, and the closed form expression for matrix elements of α is derived. In Chapter 3 the results of our calculation are presented, and in Chapter 4 we discuss the validity of this approach.

CHAPTER 2. THEORY - THE UNIFIED MODEL

The Hamiltonian

In the unified model of nuclear structure the Hamiltonian is taken to be that of a core, plus one or more nucleons moving in single particle orbitals outside this core. In addition there are residual interactions between pairs of particles, and an interaction between the core and each particle. So, one writes for the Hamiltonian

$$H = H_c + H_{sp} + H_{res} + H_{cp} \qquad 2.1$$

in which H_c is the core Hamiltonian, H_{sp} is the single particle Hamiltonian, H_{res} is the residual interaction, and H_{cp} is the coupling between the core and each valence nucleon.

The core Hamiltonian may be taken to have various collective modes (61) depending on the deformation of the core in the region under consideration. Near closed shells nuclei are known to exhibit some characteristics of a vibrational nature (62, 63), so one takes

$$\begin{aligned} H_c &= T + V \\ &= \frac{1}{2} B \sum_{\mu} \dot{\alpha}^{\mu} \dot{\alpha}_{\mu} + \frac{1}{2} C \sum_{\mu} \alpha^{\mu} \alpha_{\mu} \end{aligned} \qquad 2.2$$

This is the form for small oscillations about a spherical equilibrium (64), where the α_{μ} are collective coordinates describing the nuclear surface in the laboratory frame which is defined by (65)

$$R(\theta,\phi) = R_o[1 + \sum_{\lambda\mu} (\alpha_{\lambda\mu})^{*} Y_{\lambda\mu}(\theta,\phi)] \qquad 2.3$$

We shall consider only the lowest order vibrations of interest, $\lambda = 2$ (5).

The eigenvalue problem for this Hamiltonian was only recently completely solved to give exact, closed form, multiplicity resolved eigenfunctions, valid for an arbitrary number of phonons. These results are presented elsewhere (60), and we shall make use of them frequently as the need arises.

The eigenfunctions for the core were found to be labeled completely in the subgroup chain of the Hamiltonian's symmetry group, U(5) (60).

$$U(5) \supset SU(5) \supset R(5) \supset R(3) \supset R(2) \qquad 2.4$$

by the labels

$$|N,\ell,\nu,R,M\rangle$$

where

N = the number of phonons
  = the U(5) IR label, (and SU(5))

ℓ = R(5) IR label

ν = a label to resolve the remaining multiplicity

R = the physical R(3) IR label (angular momentum)

M = the projection of R onto the lab z-axis

These states satisfy (60)

$$H_c|N\ell\nu RM\rangle = \hbar\omega(N + \frac{5}{2})|N\ell\nu RM\rangle$$

$$A|N\ell\nu RM\rangle = \frac{1}{2}\ell(\ell + 3)|N\ell\nu RM\rangle \qquad 2.5$$

$$R^2|N\ell\nu RM\rangle = R(R+1)|N\ell\nu RM\rangle$$

$$M|N\ell\nu RM\rangle = M|N\ell\nu RM\rangle$$

(The form of A is not important here, but is discussed in (66, 67).)

For H_{sp}, the single particle Hamiltonian, we take the single particle energies as input, i.e., for 2 particles

$$H_{sp}|(j_1j_2)JM\rangle_A = (E_{j_1} + E_{j_2})|(j_1j_2)JM\rangle_A \qquad 2.6$$

In Equation 2.6 the A stands for antisymmetrization of the coupled states.

$$|(j_1j_2)J\rangle \equiv |(n_1\ell_1j_1)(n_2\ell_2j_2)JM\rangle$$

$$= [|n_1\ell_1j_1\rangle \otimes |n_2\ell_2j_2\rangle]^J_M \qquad 2.7$$

in which (68)

$$|n(\ell_1\ \tfrac{1}{2})JM\rangle = R_{n\ell}(r)[Y_\ell(\theta_i)\otimes\chi_{\frac{1}{2}}]^J_M \qquad 2.8$$

Then,

$$|(j_1j_2)JM\rangle_A = \frac{1}{\sqrt{2}[(1+\delta_{j_1j_2})]^{\frac{1}{2}}}\{|j_1(1)j_2(2);JM\rangle - |j_1(2)j_2(1);JM\rangle\} \qquad 2.9$$

We will drop the use of the subscript "A" for antisymmetrization, it being understood that we are working with antisymmetric states. The single-particle energies are either taken from experiment or from a theoretical calculation of the core + 1 particle (hole) nuclei; we take the latter course. Note that these energies once determined are fixed in our calculation; they are not free parameters.

Next we consider the form for H_{cp}. Physically, we have a nuclear surface described by the collective coordinate α, and a particle (hole) orbiting around it, whose angular dependence is described by $Y_2(\theta_i)$. The lowest order rotational scalar one can form is then

$$[\alpha \otimes Y_2]^0_0$$

So to lowest order in α (for two valence nucleons)

$$H_{cp} = \overline{+} K(r) \sum_{i=1}^{2} \sum_{\mu} \alpha^{\mu} Y_{2\mu}(\theta_i) + \ldots \qquad 2.10$$

where $\overline{+}$ refers to particles and holes respectively (27).

The radial dependence is usually ignored. We give a short argument to justify this, following Eisenberg and Greiner (61).

Assuming that the nuclear surface is an equipotential surface of the nuclear field, and given a spherical shell model potential $V_o(r,\underset{\sim}{\ell},\underset{\sim}{s})$ then for the deformed field take

$$V(\alpha_{2\mu},r,\underset{\sim}{\ell},\underset{\sim}{s}) = V\left(\frac{r}{1+\Sigma\alpha_2{}^{\mu}Y_{2\mu}},\underset{\sim}{\ell},\underset{\sim}{s}\right) \qquad 2.11$$

This has an equipotential for

$$r = R = R_o(1 + \sum_{\mu} \alpha_2{}^{\mu}Y_{2\mu})$$

Suppose we make a Taylor's series expansion in $\alpha_2{}^{\mu}$ of V

$$V = V(o,r,\underset{\sim}{\ell},\underset{\sim}{s}) + \sum_{\mu} \left[\frac{\partial V}{\partial \alpha^{\mu}}\right] \alpha^{\mu} + \ldots$$

$$= V_o(r,\underset{\sim}{\ell},\underset{\sim}{s}) + \sum_\mu \left[\frac{dV_o(r,\ell,s)}{dr}\right] \left[\frac{\partial}{\partial\alpha^\mu}\left(\frac{r}{1+\sum_\mu \alpha^\mu Y_{2\mu}}\right)\right] \alpha^\mu + \ldots$$

$$= V_o(r,\underset{\sim}{\ell},\underset{\sim}{s}) + r\left(\frac{dV}{dr}\right) \sum_\mu \alpha^\mu Y_{2\mu} + \ldots \qquad 2.12$$

Since $V_o(r,\underset{\sim}{\ell},\underset{\sim}{s})$ usually has a Woods-Saxon form,

$$\frac{dV}{dr} \sim 0 \quad \text{except near surface}$$

So we have

$$H_{cp} = -K(r) \sum_\mu \alpha^\mu Y_{2\mu} \qquad 2.13$$

Because $\frac{dV}{dr}$ is expected to vary only near the surface, K(r) will be constant over the range for which it is non-zero, and the r dependence is thus neglected (32).

The remaining term in our Hamiltonian is H_{res}, the residual interaction between pairs of valence nucleons. There have been numerous forms used by different authors in calculations of this type. The simplest form, and surprisingly enough the most successful, is the simple pairing force between pairs of particles coupled to spin zero. This form was first proposed by Kisslinger and Sorensen in a spherical shell model calculation (69). This form is written (64)

$$\langle (j_1' j_2')J' || H_{res} || (j_1 j_2)J\rangle = -G\delta_{J,J'}\delta_{j_1',j_2}\delta_{j_1,j_2} \times \sqrt{(j_1+\tfrac{1}{2})(j_1'+\tfrac{1}{2})} \qquad 2.14$$

and good agreement is found for many nuclei using

$$G = 27/A \text{ MeV}$$

The other type of residual interaction we consider is at the opposite end of the complexity spectrum. We use matrix elements calculated from a "bare" G matrix realistic interaction (70). In the calculation by Vary, the Brueckner G-matrix appropriate for medium-heavy nuclei was obtained from the Reid soft-core nucleon-nucleon potential. Our reason for using such a force was the hope that the vibrational core + valence nucleons and core-particle interaction would effectively "sum" the higher order terms in the G-matrix perturbation expansion. To the best of our knowledge, this is the first such investigation.

The Hamiltonian, Equation 2.1, is then to be diagonalized in the coupled basis

$$|(j_1 j_2)J;N\ell\nu R;IM\rangle$$

$$= \sum_{M_J M_R} C(JRI;M_J,M_R,M)\,|(j_1 j_2)JM_J\rangle|N\ell\nu RM_R\rangle \qquad 2.15$$

The core and single particle pieces are of course already diagonal

$$\langle(j_1 j_2)J;N,\ell,\nu\ R;I,M|H_c + H_{sp}|(j_1 j_2)J;N,\ell\nu R;IM\rangle$$

$$= \{\hbar\omega(N + 5/2) + E_{j_1} + E_{j_2}\} \qquad 2.16$$

The residual interaction is then found by using the form for an operator in a coupled space operating on space (1), Equation A.11. First, where it is not ambiguous, we abbreviate the notation to

$$|(j_1 j_2)J;N\ell\nu R;IM\rangle \equiv |J;NR;IM\rangle \qquad 2.17$$

Then for the residual forces, $H_{res} = T_o(1)$ (it is a rotational scalar)

$$\langle J';N'R';I'||H_{res}||J;NR;I\rangle$$

$$= (-1)^{R+O+J'+I}\hat{J}'\hat{I}\begin{Bmatrix} J & J' & O \\ I' & I & R \end{Bmatrix}\langle J'||H_{res}||J\rangle\,\delta_{N'N}\,\delta_{R'R} \qquad 2.18$$

where $\hat{j} = \sqrt{2j+1}$. But,

$$\begin{Bmatrix} J & J' & O \\ I' & I & R \end{Bmatrix} = \frac{(-1)^{J+I'+R}}{\hat{J}\hat{I}}\,\delta_{JJ'}\,\delta_{II'}\,,$$

So we have simply

$$= \langle J'||H_{res}||J\rangle\,\delta_{J'J}\,\delta_{I'I}\,\delta_{R'R}\,\delta_{N'N} \qquad 2.19$$

where $\langle J'||H_{res}||J\rangle$ is given by Equation 2.14.

Finally we consider the core-particle coupling, which we note is a scalar formed from two rank two tensors (with respect to rotation.) Using Equation A.10 for a scalar between coupled basis states we have

$$\langle J';N'R';I'||\sum_{\mu,i}\alpha^{\mu}Y_{2\mu}(i)||J;NR;I\rangle$$

$$= \delta_{I'I}\,(-1)^{J+R'+I}\hat{J}'\hat{R}'\begin{Bmatrix} J & J' & 2 \\ R' & R & I \end{Bmatrix}$$

$$\times\,\langle(j_1'j_2')J'||Y_2(1)+Y_2(2)||(j_1j_2)J\rangle$$

$$\times \langle N' \ell' \nu' R' || \alpha || N \ell \nu R \rangle \qquad 2.20$$

To evaluate the particle part we will write the matrix element between antisymmetrized states in terms of the non-antisymmetrized states. We use the notation

$$|[j_1(1)j_2(2)]JM\rangle = \frac{1}{[2(1+\delta_{j_1 j_2})]^{\frac{1}{2}}}\{|j_1(1)j_2(2);JM) - |j_1(2)j_2(1);JM)\} \qquad 2.21$$

where

$$|j_1(1)j_2(2);JM) = \sum_{M_1 M_2} c(j_1 j_2 J; M_1 M_2 M)|j_1(1)\rangle|j_2(2)\rangle \qquad 2.22$$

First we can write

$$\langle [j_1'(1)j_2'(2)]J' || Y_2(1) + Y_2(2) || [j_1(1)j_2(2)]J\rangle$$

$$= 2\langle [j_1'(1)j_2'(2)]J' || Y_2(1) || [j_1(1)j_2(2)]J\rangle$$

$$= \frac{1}{[(1+\delta_{j_1 j_2})(1+\delta_{j_1' j_2'})]^{\frac{1}{2}}}$$

$$\times \{(j_1'(1)j_2'(2)J' || Y_2(1) || j_1(1)j_2(2)J)$$

$$- (j_1'(1)j_2'(2)J' || Y_2(1) || j_1(2)j_2(1)J)$$

$$- (j_1'(2)j_2'(1)J' || Y_2(1) || j_1(1)j_2(2)J)$$

$$+ (j_1'(2)j_2'(1)J' || Y_2(1) || j_1(2)j_2(1)J)\} \qquad 2.23$$

$$= \frac{1}{[(1+\delta_{j_1 j_2})(1+\delta_{j_1' j_2'})]^{\frac{1}{2}}} \times \{(j_1'(1)j_2'(2);J'||Y_2(1)||j_1(1)j_2(2);J)$$

$$- (-1)^{j_1+j_2-J}(j_1'(1)j_2'(2);J'||Y_2(1)||j_2(1)j_1(2);J)$$

$$- (-1)^{j_1'+j_2'-J'}(j_2'(1)j_1'(2);J'||Y_2(1)||j_1'(1)j_2(2);J)$$

$$+ (-1)^{j_1'+j_2'+j_1+j_2-J-J'}(j_2'(1)j_1'(2);J'||Y_2(1)||j_2(1)j_1(2);J)\} \quad 2.24$$

Now each term has the same basic form, which we evaluate next. Using Equation A.11 for a tensor operator in space (1)

$$(j_1'(1)j_2'(2);J||Y_2(1)||j_1(1)j_2(2);J)$$

$$= (-1)^{j_2+2+j_1'+J}\hat{j}_1'\hat{J}\begin{Bmatrix} j_1 & j_1' & 2 \\ J' & J & j_2 \end{Bmatrix}\langle j_1'||Y_2||j_1\rangle\delta_{j_2' j_2} \quad 2.25$$

Using this last result we find

$$\langle(j_1'j_2')J'||\sum_{i=1}^{2}Y_2(i)||(j_1j_2)J\rangle$$

$$= \left[\frac{(2J+1)}{(1+\delta_{j_1j_2})(1+\delta_{j_1'j_2'})}\right]^{\frac{1}{2}}$$

$$\times\left\{(-1)^{j_1'+j_2+J}\hat{j}_1'\langle j_1'||Y_2||j_1\rangle\begin{Bmatrix} j_1 & j_1' & 2 \\ J' & J & j_2 \end{Bmatrix}\delta_{j_2' j_2}\right.$$

$$+ (-1)^{J+J'+1}\langle\hat{j}_2'\, j_2'||Y_2||j_1\rangle \begin{Bmatrix} j_1 & j_2' & 2 \\ J' & J & j_2 \end{Bmatrix} \delta_{j_2 j_1'}$$

$$+ (-1)^{j_1'+j_2}\, \hat{j}_1'\langle j_1'||Y_2||j_2\rangle \begin{Bmatrix} j_2 & j_1' & 2 \\ J' & J & j_1 \end{Bmatrix} \delta_{j_2' j_1}$$

$$+ (-1)^{j_1'+j_2+J'}\, \hat{j}_2'\langle j_2'||Y_2||j_2\rangle \begin{Bmatrix} j_2 & j_2' & 2 \\ J' & J & j_1 \end{Bmatrix} \delta_{j_1 j_1'} \Bigg\} . \qquad 2.26$$

Next we need

$$\langle j'||Y_2||j_1\rangle \equiv \langle(\ell',\tfrac{1}{2})j'||Y_2||(\ell,\tfrac{1}{2})j\rangle$$

$$= (-1)^{\frac{1}{2}+2+\ell'+j}\, \hat{\ell}'\hat{j} \begin{Bmatrix} \ell & \ell' & 2 \\ j' & j & \frac{1}{2} \end{Bmatrix} \langle\ell'||Y_2||\ell\rangle \qquad 2.27$$

The orbital matrix element is found by direct integration (71)

$$\langle\ell' m'|Y_{LK}|\ell m\rangle = \int_\Omega Y^*_{\ell' m'}(\theta_i) Y_{LK}(\theta_i) Y_{\ell m}(\theta_i) d\Omega$$

$$= \left[\frac{(2\ell+1)(2L+1)}{4\pi(2\ell'+1)}\right]^{\frac{1}{2}} c(\ell L\ell';m,K',m')c(\ell L\ell';000)$$

$$\equiv c(\ell L\ell', mKm')\langle\ell'||Y_L||\ell\rangle \qquad 2.28$$

so

$$\langle\ell'||Y_L||\ell\rangle = \frac{\hat{\ell}\hat{L}}{\sqrt{4\pi}\,\hat{\ell}'} c(\ell L\ell';000) \qquad 2.29$$

Putting this back into Equation 2.27 one has

$$\langle(\ell',\tfrac{1}{2})j'||Y_L||(\ell,\tfrac{1}{2})j\rangle = (-1)^{L+\frac{1}{2}+\ell'+j}\,\hat{\ell}'\hat{j}\,\frac{\hat{\ell}\hat{L}}{\sqrt{4\pi}\,\hat{\ell}'}\,c(\ell L\ell';000)\begin{Bmatrix}\ell & \ell' & 2\\ j' & j & \frac{1}{2}\end{Bmatrix}.$$

This result can be further simplified by using (68)

$$\begin{pmatrix}\ell & L & \ell'\\ 0 & 0 & 0\end{pmatrix}\begin{Bmatrix}j' & j & L\\ \ell & \ell' & \frac{1}{2}\end{Bmatrix} = -\frac{1}{\hat{\ell}\hat{\ell}'}\begin{pmatrix}j' & j & L\\ \frac{1}{2} & -\frac{1}{2} & 0\end{pmatrix}\frac{[1+(-1)^{\ell+\ell'+L}]}{2} \qquad 2.30$$

to give us

$$\langle(\ell',\tfrac{1}{2})j'||Y_L||(\ell,\tfrac{1}{2})J\rangle = \frac{\hat{L}}{\sqrt{4\pi}}\,c(j'Lj;\tfrac{1}{2}0\tfrac{1}{2})\,\frac{[1+(-1)^{\ell+\ell'+L}]}{2} \qquad 2.31$$

The remaining quantity to be evaluated is

$$\langle N'\ell'\nu'R'M'|\alpha_\mu|N\ell\nu RM\rangle$$

which we consider in the next section.

Matrix Elements of α

To evaluate the core-particle coupling matrix elements we saw we needed the reduced matrix elements of the collective coordinate α between core basis states, i.e.,

$$\langle N'\ell'\nu'R'||\alpha||N\ell\nu R\rangle$$

For $N \leq 3$ there is no multiplicity problem, and the angular momentum completely determines the states. In this case it is convenient to write the states in terms of the creation and annihilation operators b, b^+ defined by (4)

$$\underset{\sim}{b}^{+} \equiv \frac{1}{\sqrt{2}} (\underset{\sim}{\alpha}' - i\underset{\sim}{\pi}')$$

$$\underset{\sim}{b} \equiv \frac{1}{\sqrt{2}} (\underset{\sim}{\alpha}' + i\underset{\sim}{\pi}') \qquad 2.32$$

where we have introduced the dimensionless forms of α and its conjugate momentum π

$$\underset{\sim}{\alpha}' = \left[\frac{B\omega}{\hbar}\right]^{\frac{1}{2}}$$

$$\underset{\sim}{\pi}' = \left[\frac{1}{B\hbar\omega}\right]^{\frac{1}{2}} \qquad 2.33$$

and

$$\omega = \left(\frac{C}{B}\right)^{\frac{1}{2}}$$

These operators satisfy the usual creation-annihilation commutation relationships for boson operators.

The dimensionless quantities are convenient to work with, and hereafter we shall use the scaled quantities and omit the primes unless otherwise explicitly stated.

In terms of the b and b^{+} the Hamiltonian is written as

$$H = b^{+}_{\mu} b^{\mu} + \frac{5}{2} \qquad 2.34$$

and the lowest states $|NRM\rangle$ can be written (65) as

$$|12M\rangle = b^{+}_{M}|o\rangle$$

$$|2JM\rangle = \frac{1}{\sqrt{2}} \sum_{m} c(22J;m,M-m,M) b^{+}_{m} b^{+}_{M-m} \qquad 2.35$$

Using these we may find matrix elements of b^+ directly

$$\langle 2J'M'|b^+_\nu|12M\rangle$$

$$= \frac{1}{\sqrt{2}} \sum_m c(22J;m,M'-m,M')\langle o|b^m b^{M'-m} b^+_\nu b^+_M|o\rangle$$

$$= \frac{1}{\sqrt{2}} \sum_m c(22J;m,M'-m,M') \left\{ \delta_{m,M}\,\delta_{M'-m,\nu} + \delta_{m,\nu}\,\delta_{M'-m,M} \right\}$$

$$= \frac{1}{\sqrt{2}} \{c(22J;M,\nu M') + c(22J;\nu,M,M')\}$$

$$= \sqrt{2}\, c(22J;M,\nu,M')$$

where we have used the commutation relations

$$\left[b^+_\mu, b^\nu\right] = \delta^\nu_\mu \qquad 2.36$$

(The transformation and commutation properties are discussed in detail in Corrigan *et al.* (60).)

From this we see

$$\langle 2J'||b^+||12\rangle = \sqrt{2}$$

For calculations such as needed in the unified model, the standard reference for these matrix elements is the table in Choudhury's article (23), which gives matrix elements of b for $N \leq 3$. At $N = 4$ the multiplicity which is resolved by R(5) decomposition occurs, and at $N = 6$ the multiplicity we resolved by the label ν occurs (see Table I). Using our multiplicity resolved eigenfunctions (60), we may arrive at a closed form expression valid for any N.

Table I. U(5) basis states for N less than six. The values of ℓ and ν for a given N and R are shown

						(ℓ,ν)							
N \ R	0	1	2	3	4	5	6	7	8	9	10	11	12
0	(0,0)												
1			(1,0)										
2	(0,0)		(2,0)		(2,0)								
3	(3,1)		(1,0)	(3,0)	(3,0)		(3,0)						
4	(0,0)		(4,1) (2,0)		(4,0) (2,0)	(4,0)	(4,0)		(4,0)				
5	(3,1)		(1,0) (5,1)	(3,0)	(3,0) (5,1)	(5,0)	(3,0) (5,0)	(5,0)	(5,0)		(5,0)		
6	(2,0) (0,0)		(2,0) (4,1)	(6,1)	(2,0) (4,0) (6,1)	(4,0)	(4,0) (6,1) (6,0)	(6,0)	(4,0) (6,0)	(6,0)	(6,0)		(6,0)

The eigenfunctions were found to have the form

$$\Psi_{N\ell\nu RM}(\beta,\gamma,\theta_i)$$

$$= f_{N\ell}(\beta)\eta_{N\ell\nu} \sum_{\substack{K\geq 0 \\ \text{even}}} \frac{1}{[1+\delta_{K,0}]} g_{\ell\nu JK}(\gamma) \left[D^J_{M,K}{}^*(\theta_i)+(-1)^J D^J_{M,-K}{}^*(\theta_i) \right] \qquad 2.37$$

These are written in terms of body-fixed frame quantities β, γ, θ_i, where the lab to body transformation is

$$a_\mu = \sum_\nu D^2_{\nu\mu}(\theta_i)\alpha_\mu \qquad 2.38$$

The nuclear surface in the body frame is then written as

$$R(\theta',\phi') = R_o[1 + \sum_\mu a^\mu Y_{2\mu}(\theta',\phi')] \qquad 2.39$$

These body coordinates are required to satisfy (65)

$$a_1 = a_{-1} = 0$$

$$a_2 = a_{-2} \qquad \text{(real)} \qquad 2.40$$

$$a_0 \qquad \text{(real)}$$

The usual choice of generalized body-frame coordinates is $\{\beta,\gamma,\theta_i\}$ where the θ_i are defined by Equation 2.38 as the Euler angles of orientation of the body frame relative to the fixed laboratory frame (that is, they specify the rotation necessary to take the lab to the body frame). Then β and γ are defined by

$$a_0 = \beta \cos \gamma$$
$$a_{\pm 2} = \frac{\beta}{\sqrt{2}} \sin \gamma \qquad 2.41$$

where β is called the deformation, and γ the asymmetry parameter. If $\gamma = 0$, and $\beta = 0$ the surface is a sphere; if $\gamma = 0$, $\beta \neq 0$ the surface is a spheroid (ellipse rotated about the major or minor axis); if $\gamma \neq 0$ and $\beta \neq 0$ the surface is an ellipsoid (no symmetry axis).

We use these eigenfunctions to evaluate matrix elements of α by direct integration. We have

$$\int_0^\infty d\beta \int_0^{2\pi} |\sin 3\gamma| d\gamma \int_\Omega d\Omega \left\{ \Psi^*_{N\ell\nu RM}(\beta,\gamma,\theta_i) \alpha_\mu \Psi_{N\ell\nu RM}(\beta,\gamma,\theta_i) \right\} \qquad 2.42$$

When we use Equation 2.38 to write α_μ in terms of body fixed coordinates, the $D^2_{\mu\nu}(\theta_i)$ will appear, and with a rotation matrix from each Ψ term we simply perform the integral over three rotation matrices, which contributes only a pair of Clebsch Gordan coefficients. The β and γ integrals will then remain.

We first note that the U(5) eigenfunctions factor into a radial and an angular part (60) corresponding to the decomposition $U(5) \supset SO(2,1) \times R(5)$ as

$$|N\ell\nu RM\rangle = |N\ell\rangle |\ell\nu RM\rangle \qquad 2.43$$

where the radial functions $|N\ell\rangle$ are Laguerre polynomials in the radial variable β. We can evaluate the radial part separately. We want

$$\langle N' \ell' | \beta | N \ell \rangle \qquad 2.44$$

given (60)

$$\langle \beta | N \ell \rangle = f_{n\ell}(\beta)$$

$$= (-1)^n n!\ \beta^{\ell} e^{-\beta^2/2}\ L_n^{\ell+3/2}(\beta^2)\ N(n\ell) \qquad 2.45$$

where

$$N = 2n + \ell \qquad N(n,\ell) = \left[\frac{2(n!)}{\Gamma(n+\ell+5/2)} \right]^{\frac{1}{2}} \qquad 2.46$$

The selection rules can be obtained easily by using the Wigner-Eckart theorem for SO(2,1) as in Wybourne (72). First one shows that r^K is a tensor of rank K/2 in SO(2,1). The states are written

$$|n\ell\rangle = |Tt\rangle \qquad 2.47$$

where

$$T = \tfrac{1}{2}(\ell - \tfrac{1}{2}) \quad , \quad t = \tfrac{1}{2}(n + 3/2) \qquad 2.48$$

and

$$\underset{\sim}{K}^2 |Tt\rangle = T(T+1)|Tt\rangle$$
$$K_o |Tt\rangle = t|Tt\rangle \qquad 2.49$$

Here, $K_{\pm}$, K_o generate SO(2,1), and are defined by

$$K_+ = \tfrac{1}{2}(\underset{\sim}{b}^+ \cdot \underset{\sim}{b}^+)$$
$$K_- = \tfrac{1}{2}(\underset{\sim}{b} \cdot \underset{\sim}{b}) \qquad 2.50$$

$$K_o = \frac{H}{2} \quad .$$

The Wigner-Eckart theorem (72) then asserts that

$$\langle T' t' | T(\Lambda\lambda) | Tt \rangle = \bar{c}(T\Lambda T'; t\lambda t') \langle T' || T(\Lambda) || T \rangle$$

where the SO(2,1) Clebsch-Gordan coefficients $\bar{c}$ differ from those of SO(3) by only a phase factor (72). Thus, we have the selection rules

$$\Delta(T'\Lambda T)$$

$$t' = t + \lambda \quad . \qquad 2.51$$

Here,

$$\Lambda = \tfrac{1}{2} \qquad \lambda = \pm\tfrac{1}{2}$$

so that

$$t' = t \pm \tfrac{1}{2} \qquad 2.52$$

or in terms of n

$$\tfrac{1}{2}(n' + 3/2) = \tfrac{1}{2}(n + 3/2) \pm \tfrac{1}{2}$$

$$n' = n \pm 1 \quad . \qquad 2.53$$

Similarly

$$T' = T \pm \tfrac{1}{2}$$

so that

$$\ell' = \ell \pm 1 \quad . \qquad 2.54$$

We could continue and derive matrix elements this way, but we chose a more straightforward approach.

We use a basic property of Laguerre polynomials (73)

$$XL_n^{\alpha}(X) = (\alpha + n)L_n^{\alpha-1}(X) - (n + 1)L_{n+1}^{\alpha-1}(X) \qquad 2.55$$

to find

$$\beta|N\ell\rangle = \beta\, f_{n\ell}(\beta)$$

$$= N(n\ell)\beta^{\ell-1}e^{-\beta 2/2}\left[\beta^2 L_n^{\ell+3/2}(\beta^2)\right]\ (-1)^n$$

$$= N(n\ell)\beta^{\ell-1}e^{-\beta 2/2}\left\{(n+\ell+5/2)L_n^{(\ell+1)+5/2} - (n+1)L_{n+1}^{(\ell+1)+5/2}\right\}(-1)^n$$

$$= (n+\ell+3/2)\,\frac{N(n,\ell)}{N(n,\ell-1)}\,|n,\ell-1\rangle + (-1)^n(n+1)\,\frac{N(n,\ell)}{N(n+1,\ell-1)}\,|n+1,\ell-1\rangle \quad . \qquad 2.56$$

Hence, it follows that

$$\langle n+1,\ell-1|\beta|n,\ell\rangle = (n+1)\,\frac{N(n,\ell)}{N(n+1,\ell-1)}$$

$$= (n+1)\left[\frac{2(n!)\Gamma(n+\ell+5/2)}{\Gamma(n+\ell+5/2)2(n+1!)!}\right]^{\frac{1}{2}}$$

$$= \sqrt{n+1} \qquad 2.57$$

and

$$\langle n,\ell-1|\beta|n,\ell\rangle = (n+\ell+3/2)\,\frac{N(n,\ell)}{N(n,\ell-1)}$$

$$= (n+\ell+3/2)\left[\frac{2\,n!}{\Gamma(n+\ell+5/2)}\,\frac{\Gamma(n+\ell+3/2)}{2\,n!}\right]^{\frac{1}{2}}$$

$$= (n+\ell+3/2)\left[\frac{1}{(n+\ell+3/2)}\right]^{\frac{1}{2}}$$

$$= \sqrt{n+\ell+3/2}\ . \qquad 2.58$$

Then, using

$$\langle n'\ell'|\beta|n\ell\rangle = \langle n\ell|\beta|n'\ell'\rangle \qquad 2.59$$

we find

$$\langle n-1,\ell+1|\beta|n,\ell\rangle = \langle n,\ell|\beta|n-1,\ell+1\rangle$$

$$= \sqrt{n} \qquad 2.60$$

$$\langle n,\ell+1|\beta|n,\ell\rangle = \langle n\ell|\beta|n,\ell+1\rangle$$

$$= \sqrt{n+\ell+5/2}\ . \qquad 2.61$$

These results are summarized in terms of the U(5) label N as:

$$\begin{aligned}
\langle N+1,\ell-1|\beta|N,\ell\rangle &= \frac{1}{\sqrt{2}}\,[N-\ell+2]^{\frac{1}{2}} \\
\langle N-1,\ell-1|\beta|N,\ell\rangle &= \frac{1}{\sqrt{2}}\,[N+\ell+3]^{\frac{1}{2}} \\
\langle N-1,\ell+1|\beta|N,\ell\rangle &= \frac{1}{\sqrt{2}}\,[N-\ell]^{\frac{1}{2}} \\
\langle N+1,\ell+1|\beta|N,\ell\rangle &= \frac{1}{\sqrt{2}}\,[N+\ell+5]^{\frac{1}{2}}\ .
\end{aligned} \qquad 2.62$$

We now evaluate the R(5) piece, doing the integrals over γ and θ_i explicitly. The forms for the eigenfunctions were shown in Equation 2.37. We have

$$\langle \ell' \nu' R' M' | \frac{\alpha_\mu}{\beta} | \ell \nu J R M \rangle$$

$$= \int_0^{2\pi} |\sin 3\gamma| \int_\Omega \phi^*_{\ell' \nu' R' K'}(\gamma,\theta_i) \frac{\alpha_\mu}{\beta} \phi_{\ell \nu R K}(\gamma,\theta_i) d\gamma d\Omega$$

$$= \int_0^{2\pi} |\sin 3\gamma| \int_\Omega \left\{ N_{\ell' \nu' R'} \sum_{\substack{K' \geq 0 \\ \text{even}}} \frac{1}{[1+\delta_{K',0}]} g_{\ell' \nu' R' K'}(\gamma) \right.$$

$$\left. \left[D^{R'}_{M',K'}(\theta_i) + (-1)^{R'} D^{R'}_{M',-K'}(\theta_i) \right] \right\}$$

$$\times \frac{\alpha_\mu(\beta,\gamma,\theta_i)}{\beta}$$

$$\times \left\{ N_{\ell \nu R} \sum_{\substack{K \geq 0 \\ \text{even}}} \frac{1}{[1+\delta_{K,0}]} g_{\ell \nu RK}(\gamma) \left[D^{R\,*}_{M,K}(\theta_i) + (-1)^R D^{R\,*}_{M,-K}(\theta_i) \right] \right\} \qquad 2.63$$

Using the inverse of Equation 2.38 to write α in body coordinates we have

$$\alpha_\mu = \sum_\rho D^{2*}_{\mu\rho}(\theta_i) a_\rho \quad . \qquad 2.64$$

The integration over Euler angles is then (71)

$$\int_{\Omega} d\Omega \left[D^{R'}_{M',K'}(\theta_i) + (-1)^{R'} D^{R'}_{M',-K'}(\theta_i) \right] D^{2*}_{\mu\rho}(\theta_i) \left[D^{R*}_{MK}(\theta_i) + (-1)^{R} D^{R\ *}_{M,-K}(\theta_i) \right]$$

$$= \frac{1}{(2R'+1)} c(R2R';M,\mu,M')$$

$$\times \{c(R2R';K,\rho,K') + (-1)^{R} c(R2R';-K,\rho,K)$$

$$+ (-1)^{R'} c(R2R';K,\rho,-K') + (-1)^{R+R'} c(R2R';-K,\rho,-K')\} \qquad 2.65$$

Then using (60)

$$g_{\ell\nu RK}(\gamma) = \sum_{f=0}^{\left[\frac{\ell-\rho}{2}\right]-m} A_f(\ell\nu RK)(\cos\gamma)^{\ell-2f-K/2}(\sin\gamma)^{2f+K/2} \qquad 2.66$$

the γ-integral is found to be of the form

$$\int_0^{2\pi} |\sin 3\gamma| \ (\cos\gamma)^{p'} (\sin\gamma)^{q'} d\gamma \qquad 2.67$$

where

$$p' = \ell + \ell' - 2(f + f') - \tfrac{1}{2}(K + K') + \delta_{\rho,0}$$
$$q' = 2(f + f') + \tfrac{1}{2}(K + K') + \delta_{\rho,\pm 2} \ . \qquad 2.68$$

This is just the integral which arose during normalization, generalized to both odd and even powers of cos and sin. However, one can

show by symmetry arguments that this integral vanishes over the range $[0,2\pi]$ unless both p' and q' are even, giving us

$$\mathcal{Y}(p,q) = \int_0^{2\pi} |\sin 3\gamma| \, (\cos\gamma)^{2p} (\sin\gamma)^{2q} d\gamma$$

$$= 4 \sum_{n=0}^{q+1} \binom{q+1}{n} \frac{(-1)^{n+1}}{(2n+2p+1)} \left\{ \frac{3n+3q+1}{q+1} \right\} \left\{ \frac{2^{2n+2p}-1}{2^{2n+2p}} \right\} \quad . \qquad 2.69$$

Pulling these results together we have

$$\langle \ell' \nu' R' || \alpha || \ell \nu R \rangle$$

$$= \eta_{\ell' \nu' R' \, \ell \nu R} \sum_{K' \geq 0 \text{ even}} \sum_{K \geq 0 \text{ even}} \sum_{p=-2 \text{ even}} \left\{ \frac{1}{(2R'+1)} \, \frac{1}{\sqrt{2}\delta_{|\rho|2}+\delta_{\rho,0}} \right.$$

$$\frac{1}{\left[(1+\delta_{K,0})(1+\delta_{K',0}) \right]}$$

$$\times \left[c(R2R';K\rho K') + (-1)^{R} c(R2R';-K\rho K') \right.$$

$$\left. + (-1)^{R'} c(R2R';K\rho K') + (-1)^{R+R'} c(R2R';-K\rho-K') \right]$$

$$\times \sum_{f=0}^{\left[\frac{\ell-p}{2}\right]-m} \sum_{f'=0}^{\left[\frac{\ell'-p'}{2}\right]-m'} \left(A_f(\ell' \nu' R' K') A_f(\ell \nu R K) \right.$$

$$\left. \left. \times \mathcal{Y}\left(\tfrac{1}{2}(\ell+\ell'-2(f+f') - \frac{K+K'}{2} + \delta_{\rho,0}, \tfrac{1}{2}(2(f+f') + \frac{K+K'}{2} + \delta_{|\rho|,2})\right) \right) \right\}$$

2.70

Combined with the values for $\langle N'\ell'|\beta|N\ell\rangle$ these give us an expression for $\langle N'\ell'\nu'R'||\alpha||N\ell\nu R\rangle$. These were checked by comparison with the values for $\langle N'R'||b||NR\rangle$ given by Choudhury (23). (To do this we had to convert between his notation and ours.) Our results are shown in table II and table III. We derive a few useful results for storing these.

From equation 2.32

$$\underset{\sim}{\alpha} = \frac{1}{\sqrt{2}}(\underset{\sim}{b}^{+} + \underset{\sim}{b})$$

$$\underset{\sim}{\pi} = \frac{i}{\sqrt{2}}(\underset{\sim}{b}^{+} - \underset{\sim}{b}) \qquad 2.71$$

we have

$$\langle N+1,\ell+1,\nu',R'||\alpha||N,\ell,\nu,R\rangle = \frac{1}{\sqrt{2}}\langle N+1,\ell+1,\nu',R'||b^{+}||N,\ell,\nu,R\rangle \qquad 2.72$$

$$\langle N-1,\ell-1,\nu',R'||\alpha||N,\ell,\nu,R\rangle = \frac{1}{\sqrt{2}}\langle N-1,\ell-1,\nu',R'||b||N,\ell,\nu,R\rangle \qquad 2.73$$

It is important to note here that the reduced matrix elements are defined in terms of the contravariant (lower index) tensors under R(3), i.e.,

$$\langle J'M'|T_{\lambda,q}|JM\rangle \equiv c(J\lambda J';MqM')\langle J'||T_{\lambda}||J\rangle \qquad 2.74$$

While in principle this could have been defined in terms of the covariant quantity, once the choice is made, they are not equivalent, for example with R(3) tensors

Table II. Reduced matrix elements of α between core [U(5)] states for $N \leq 3$[a]

	|0000⟩	|1102⟩	|2000⟩	|2202⟩	|2204⟩	|3102⟩	|3303⟩	|3304⟩	|3306⟩	|3310⟩
⟨0000|	0	$\frac{5}{2}$		0					0	
⟨1102|	$\frac{1}{2}$	0	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{4}{5}$				0	
⟨2000|		1				$\frac{35}{10}$	0	0	0	0
⟨2202|	0	-1		0		$-\frac{2}{7}$	$-\frac{3}{2}$	$\frac{99}{70}$	0	$\frac{3}{10}$
⟨2204|		1				$\frac{2}{7}$	$-\frac{1}{3}$	$-\frac{10}{14}$	$\frac{13}{6}$	0
⟨3102|			$\frac{7}{10}$	$-\frac{2}{7}$	$\frac{36}{70}$					
⟨3303|			0	$\frac{15}{14}$	$\frac{6}{14}$					
⟨3304|	0	0	0	$\frac{11}{14}$	$-\frac{10}{14}$				0	
⟨3306|			0	0	$\frac{3}{2}$					
⟨3310|			0	$\frac{3}{2}$	0					

[a]All values are to be understood as $\sqrt{\quad}$.

Table III. Reduced matrix elements of α between R(5) states for $N \leq 6$ calculated using Equation 2.70

LP	NUP	RP	NU	R	<LP,NUP,RP||ALPHA||LP-1,NU,R>
1	0	2	0	0	0.44721359549995770 00
2	0	2	0	2	-0.53452248382484840 00
2	0	4	0	2	0.53452248382484840 00
3	0	3	0	2	0.48795003647426640 00
3	0	3	0	4	0.30860669924183600 00
3	0	4	0	2	0.41785544701867210 00
3	0	4	0	4	-0.39840953644479750 00
3	0	6	0	4	0.57735026918962550 00
3	1	0	0	2	0.57735026918962520 00
4	0	4	0	3	0.43885372573625570 00
4	0	4	0	4	-0.40881022918884940 00
4	0	4	0	6	-0.62593670241736390-01
4	0	5	0	3	0.43693144875265170 00
4	0	5	0	4	0.29457912265490280 00
4	0	5	0	6	0.29317300902903820 00
4	0	6	0	4	0.49792959773196840 00
4	0	6	0	6	-0.34015067152490320 00
4	0	8	0	6	0.60302268915553130 00
4	1	2	0	3	0.40824829046386320 00
4	1	2	0	4	0.25226248955475540 00
4	1	2	1	0	0.36514837167010980 00
5	0	5	0	4	0.51413584229818490 00
5	0	5	0	5	-0.30382181012509970 00
5	0	5	0	6	0.16724840200141810 00
5	0	6	0	4	0.39638692925226210 00
5	0	6	0	5	0.26063780986661900 00
5	0	6	0	6	-0.39449485673431520 00
5	0	6	0	8	-0.62725808176938790-01
5	0	7	0	5	0.51355259101309530 00
5	0	7	0	6	0.21483446221182970 00
5	0	7	0	8	0.27335924115579950 00
5	0	8	0	6	0.54065487356325030 00
5	0	8	0	8	-0.30382181012509940 00
5	0	10	0	8	0.62017367294604170 00

Table III. (Continued)

5	1	2	0	4	0.3144854510165750D 00
5	1	4	0	4	0.1771631875507964D 00
5	1	4	0	5	0.3567530340063376D 00
5	1	4	0	6	0.1764955169807540D 00
5	1	2	1	2	-0.5345224838248480D 00
5	1	4	1	2	0.4413674147523742D 00
6	0	6	0	5	0.5318126972686253D 00
6	0	6	0	6	-0.3235404598200727D 00
6	0	6	0	7	0.1063625394537097D 00
6	0	6	0	8	-0.3440691913314665D-01
6	0	7	0	5	0.3865883462425616D 00
6	0	7	0	6	0.3545621041711783D 00
6	0	7	0	7	-0.2977325790784309D 00
6	0	7	0	8	0.1902379462422742D 00
6	0	8	0	6	0.4764119286834183D 00
6	0	8	0	7	0.1802182911877468D 00
6	0	8	0	8	-0.3703592740877847D 00
6	0	8	0	10	-0.5819836618564209D-01
6	0	9	0	7	0.5527707983925673D 00
6	0	9	0	8	0.1701392618446803D 00
6	0	9	0	10	0.2559239653166383D 00
6	0	10	0	8	0.5681610841511977D 00
6	0	10	0	10	-0.2778362511554988D 00
6	0	12	0	10	0.6324553203677410 00
6	0	6	1	4	-0.2337980236761111D-13
6	1	3	0	5	0.2449489742783555D 00
6	1	4	0	5	0.2087815690853329D 00
6	1	4	0	6	0.2264554068288946D 00
6	1	6	0	5	0.1156921895477815D-01
6	1	6	0	6	0.2079527234916065D 00
6	1	6	0	7	0.3181535212570626D 00
6	1	6	0	8	0.1389041095631552D 00
6	1	3	1	2	0.4629100498863472D 00
6	1	3	1	4	0.3545621041712220D 00
6	1	4	1	2	0.3616202853397498D 00
6	1	4	1	4	-0.4175631381706670D 00
6	1	6	1	4	0.4859071961017038D 00
6	2	0	1	2	0.6324555203353750 00

$$\langle J'M'|T_\lambda^q|JM\rangle = \langle J'M'|(-1)^q T_{\lambda,q}|JM\rangle$$

$$= (-1)^q\, c(J\lambda J';M,-q,M')\langle J'||T_\lambda||J\rangle \qquad 2.75$$

We can also relate matrix elements of $b + b^+$ since they are hermitian conjugates so that

$$\langle J'M'|b_\mu|JM\rangle = \langle JM|b^{+\mu}|J'M'\rangle$$

$$= (-1)^\mu\, c(J'2J;M',-\mu,M)\langle J||b^+||J'\rangle \quad . \qquad 2.76$$

But also, we have

$$\langle J'M'|b_\mu|JM\rangle = c(J2J';M\mu M')\langle J'||b||J\rangle$$

$$= (-1)^\mu \left[\frac{2J'+1}{2J+1}\right]^{\frac{1}{2}} c(J'2J;-M'\mu-M)\langle J'||b||J\rangle$$

$$= (-1)^\mu \left[\frac{2J'+1}{2J+1}\right]^{\frac{1}{2}} (-1)^{J'-J}\, c(J'2J;M',-\mu,M)\langle J'||b||J\rangle$$

so that

$$\langle J'||b||J\rangle = (-1)^{J'-J}\left[\frac{2J+1}{2J'+1}\right]^{\frac{1}{2}}\langle J||b^+||J'\rangle \quad . \qquad 2.77$$

In terms of α, this allows us to store only matrix elements with $\ell' = \ell + 1$, since for $\ell' = \ell - 1$ we have

$$\langle \ell-1,\nu',J'||\alpha||\ell,\nu,J\rangle$$

$$= (-1)^{J'-J}\left[\frac{2J+1}{2J'+1}\right]^{\frac{1}{2}}\langle \ell\nu J||\alpha||\ell-1,\nu',J'\rangle \qquad 2.78$$

Finally, we clean up the units. In terms of the property dimensioned quantities

$$\underset{\sim}{\alpha} = \sqrt{\frac{\hbar}{B\omega}}\ \underset{\sim}{\alpha}' = \sqrt{\frac{\hbar\omega}{c}}\ \underset{\sim}{\alpha}' \qquad 2.79$$

A dimensionless coupling constant ξ is then introduced (42) by writing

$$H_{cp} = -K \sqrt{\frac{\hbar\omega}{c}} \sum_{\mu} \alpha'^{\mu} Y_{2\mu}$$

$$= -K \sqrt{\frac{\hbar\omega}{c}} \left\{ \sqrt{\frac{5}{\pi}} \right\} \sum_{\mu} \alpha^{\mu} \mathcal{Y}_{2\mu} \qquad 2.80$$

where

$$\mathcal{Y}_{2\mu} \equiv \sqrt{\frac{\pi}{5}}\ Y_{2\mu} \qquad 2.81$$

Then, one defines

$$\xi \equiv \frac{K}{\sqrt{\hbar\omega c}} \sqrt{\frac{5}{2\pi}} \qquad 2.82$$

which yields

$$H_{cp} = -\xi\hbar\omega\ \sqrt{2} \sum_{\mu} \alpha'^{\mu} \mathcal{Y}_{2\mu} \qquad 2.83$$

(The $\sqrt{2}$ shows up because historically this has been written in terms of b^{+} and b.)

Electromagnetic Transitions

The most useful quantity in the discussion of electromagnetic transitions is the reduced matrix element $B(E\lambda, i\to f)$, and $B(M\lambda; i\to f)$ defined by (65),

$$B(\lambda; i \to f) = \frac{1}{2I_i+1} \sum_{m,m_f} |\langle f|M(\lambda)|i\rangle|^2 \qquad 2.84$$

where $M(\lambda) = M(E\lambda)$ is the operator for electric multipole transitions and $M(\lambda) = M(M\lambda)$ is the operator for magnetic multipole transitions.

The transition probability per unit time for the emission of a photon with energy $\hbar\omega = \hbar ck$ and angular momentum λ is (65) given in terms of the reduced matrix elements as

$$T(\lambda) = \frac{8\pi(\lambda+1)}{\lambda[(2\lambda+1)!!]^2} \frac{K^{2\lambda+1}}{\hbar} B(\lambda; i \to f) \qquad 2.85$$

The multipole operators will each have contributions from both the core and the valence nucleons. For the electric multipole operator

$$M_\mu(E\lambda) = M_\mu(E\lambda)_{core} + M_\mu(E\lambda)_{particle} ,$$

where (assuming a uniform charge distribution) (61) one has

$$M_\mu(E\lambda)_{core} = \frac{3Ze}{4\pi R_o^3} \int_\Omega \int_0^R r^{\lambda+2} Y_{\lambda\mu}(\Omega) dr d\Omega \qquad 2.86$$

in which the integration is over the nuclear volume and (65)

$$M_\mu(E\lambda)_{particle} = \sum_{i=1}^{2} \pm e_i^{eff} r_i^2 Y_{2\mu}(\Omega_i) . \qquad 2.87$$

These operators are given in the laboratory frame.

The lowest order operator, $\lambda = 0$ corresponds to electric monopole or E(0) with selection rule $\Delta I = 0$ and no parity change.

Electric dipole transitions ($\lambda = 1$) are forbidden in this model because they would involve a parity change (62) and we treat only even parity states.

Since the transition probability decreases very rapidly with increasing multipolarity λ (64), the dominant electric transitions will be quadrupole ($\lambda = 2$). To evaluate the collective piece we use Equation 2.3 and expand in powers of $\alpha^{\mu} \equiv \alpha_2{}^{\mu}$

$$M_m(E2) = \frac{3Ze}{4\pi R_o{}^3} \int_{\Omega} \int_0^R r^4 Y_{\ell m}(\Omega) dr d\Omega$$

$$= \frac{3Ze}{4\pi R_o{}^3} \frac{R_o{}^5}{5} \int_{\Omega} [1 + \sum_{\mu} \alpha^{\mu} Y_{2\mu}]^5 Y_{2m}(\Omega) d\Omega$$

$$= \frac{3ZeR_o{}^2}{4\pi o5} \int_{\Omega} \left\{ 1 + 5 \sum_{\mu} \alpha_{\mu} Y_{2\mu}^{*} + \frac{5.4}{2} \sum_{\mu,\mu'} \alpha_2{}^{\mu} \alpha_2{}^{\mu'} \right.$$

$$\left. \times Y_{2\mu}(\Omega) Y_{2\mu'}(\Omega) + \ldots \right\} Y_{2m}(\Omega) d\Omega$$

$$= \frac{3ZeR_o{}^2}{4\pi o5} \left\{ \sqrt{4\pi}\, \delta_{2,0} \delta_{m,0} + 5\alpha_{\mu} \delta_{\mu,m} \right.$$

$$\left. + 10 \sum_{\mu,\mu'} \alpha^{\mu} \alpha^{\mu'} \sqrt{\frac{5}{4\pi}}\, c(222;m-\mu,\mu,m)\, c(222;000) + \ldots \right\}$$

2.88

so to first order in α^{μ}

$$M_{\mu}(E2)_{core} = \frac{3ZeR_o^2}{4\pi} \alpha_{2\mu} \quad . \tag{2.89}$$

Similarly for the magnetic multipole operators one has

$$M_{\mu}(M\lambda) = M_{\mu}(M\lambda)_{core} + M_{\mu}(M\lambda)_{particle}$$

where (65)

$$M_{\mu}(M\lambda)_{core} = \frac{-i}{\lambda+1} \sqrt{\frac{4\pi}{2\lambda+1}} \int (r \times \underset{\sim}{\nabla}) r^{\ell} Y_{\ell m} \cdot \underset{\sim}{j}(r) d\tau \tag{2.90}$$

which can be written in the form (for $\lambda = 1$)

$$M_{\mu}(M1)_{core} = \frac{e}{2mc} g_c R_{\mu} \sqrt{\frac{3}{4\pi}}$$

$$= \mu_o g_c R_{\mu} \sqrt{\frac{3}{4\pi}} \tag{2.91}$$

where g_c is given by

$$\rho_e = \frac{e}{m} g_c \rho_m \tag{2.92}$$

and R_{μ} is the μ^{th} component of the core angular momentum operator in the laboratory frame. Typically one expects (46) $g_c = \frac{Z}{A}$.

Because we consider identical valence particles their contribution may be written as

$$\underset{\sim}{M}(M1) = \sqrt{\frac{3}{4\pi}} \left\{ g_{\ell} \underset{\sim}{L} + g_S \underset{\sim}{S} \right\} \mu_o \tag{2.93}$$

where

$$\underset{\sim}{L} = \underset{\sim}{L}(1) + \underset{\sim}{L}(2)$$

$$\underset{\sim}{S} = \underset{\sim}{S}(1) + \underset{\sim}{S}(2)$$

and g_ℓ and g_S are the appropriate orbital and spin gyromagnetic factors respectively. The matrix elements of this operator will be evaluated in the basis in which $\underset{\sim}{J} = \underset{\sim}{L} + \underset{\sim}{S}$ is diagonal. Thus, in this model the total magnetic operator has the form

$$\underset{\sim}{M}(M1) = \sqrt{\frac{3}{4\pi}} \left\{ g_c \underset{\sim}{R} + g_\ell \underset{\sim}{L} + g_S \underset{\sim}{S} \right\} \mu_o \qquad 2.94$$

It is convenient to rewrite this in terms of the total spin I and the particle spin J

$$\underset{\sim}{M}(M1) = \sqrt{\frac{3}{4\pi}} \left\{ g_R \underset{\sim}{I} + (g_\ell - g_R)\underset{\sim}{J} + (g_S - g_\ell)\underset{\sim}{S} \right\} \mu_o \qquad 2.95$$

Next we derive the form for matrix elements of these operators. For E2 transitions the core operator has reduced matrix elements given by

$$\langle J';N'R';I'||M(E2)_{coll}||J;NR;I\rangle$$

$$= \langle J';N'R'I'||\frac{3ZeR_o^2}{4\pi}\alpha||J;NR;I\rangle$$

$$= \frac{3ZeR_o^2}{4\pi}(-1)^{R+J+I'}\hat{R}'\hat{I}\begin{Bmatrix} R & R' & 2 \\ I' & I & J \end{Bmatrix}\langle N'\ell'\nu'R'||\alpha||N\ell\nu R\rangle$$

$$\times \delta_{J'J}\,\delta_{j_1'j_1}\,\delta_{j_2'j_2}\,\delta_{\ell_1'\ell_1}\,\delta_{\ell_1'\ell_1} \qquad 2.96$$

which involves the same reduced matrix elements for α as were derived earlier. The particle term reduced matrix elements are given by

$$\langle J';N'R';I'||M(E2)_{part}||J;NR;I\rangle$$

$$= \langle J';N'R';I'||\, e^{eff}\left\{ r^2(1)Y_2(1) + r^2(2)Y_2(2)\right\} ||J;NR;I\rangle$$

$$= (-1)^{R+J'+I}\,\hat{J}'\hat{I}\begin{Bmatrix} J & J' & 2 \\ I' & I & R \end{Bmatrix}\langle N'\ell'\nu'R'|N\ell\nu R\rangle\, e^{eff}$$

$$\cdot \frac{3R_o^2}{5}\langle (j_1'j_2')J'||\sum_{i=1} Y_2(i)||(j_1j_2)J\rangle \qquad 2.97$$

where matrix elements of r^2 are set equal to $\frac{3}{5}R_o^2$ for simplicity (27). Since the radial dependence in H_{cp} was neglected, it would be inconsistent to reintroduce it here. This is the procedure used by Alga, Vanden Berghe, etc. We have already evaluated the matrix elements of $Y_2(1) + Y_2(2)$ in equation 2.26. The core eigenstates are orthonormal for $N \leq 5$, which are as much as we ever actually use. The overlaps for arbitrary states is found in section IV of reference (60). The concept of effective charge is discussed in Appendix B. We also set $R_o = 1.2\,A^{1/3}$ fm.

For the magnetic dipole operator one needs to evaluate matrix elements of $\underset{\sim}{I}$, $\underset{\sim}{J}$, and $\underset{\sim}{S}$. The first of these is trivial

$$\langle (j_1' j_2')J';N'\ell'\nu'R';I'||1||(j_1 j_2)J;N\ell\nu R;I\rangle$$

$$= \sqrt{I(I+1)}\ \delta_{I'I}\,\delta_{JJ}\,\delta_{j_1'j_1}\,\delta_{j_2'j_2}\,\langle N'\ell'\nu'R'|N\ell\nu R\rangle \qquad 2.98$$

Matrix elements of $\underset{\sim}{J}$ are almost as easy being given by

$$\langle J';N'R';I'||J||J;NR;I\rangle$$

$$= (-1)^{R+I+J'+I}\,\hat{J}'\hat{I}\begin{Bmatrix} J & J' & I \\ I' & I & R \end{Bmatrix}\langle J'||J||J\rangle\langle N'R'|NR\rangle$$

$$= (-1)^{R+I+J+I}\,\hat{J}\hat{I}\,\sqrt{J(J+1)}\ \delta_{J'J}\,\delta_{j_1'j_1}\,\delta_{j_2'j_2}\begin{Bmatrix} J & J & I \\ I' & I & R \end{Bmatrix}$$

$$\times\ \langle N'\ell'\nu'R'|N\ell\nu R\rangle \qquad 2.99$$

For matrix elements of $\underset{\sim}{S} = \underset{\sim}{S}(1) + \underset{\sim}{S}(2)$ we use essentially the same procedure we used for $Y_2(1) + Y_2(2)$, that is we expand the antisymmetrized single particle states and rewrite by recoupling in terms of matrix elements only in space (1). First we write

$$\langle J';N'R';I'||\underset{\sim}{S}||J;NR;I\rangle$$

$$= (-1)^{R+I+J'+I}\,\hat{J}'\hat{I}\begin{Bmatrix} J & J' & I \\ I' & I & R \end{Bmatrix}\langle N'R'|NR\rangle$$

$$\times\ \langle (j_1' j_2')J'||S||(j_1 j_2)J\rangle \qquad 2.100$$

Then we proceed as outlined above,

$$\langle (j_1' j_2')J' || S(1) + S(2) || (j_1 j_2)J \rangle$$

$$= \frac{1}{(1+\delta_{j_1 j_2})(1+\delta_{j_1' j_2'})^{\frac{1}{2}}} \left\{ \left((j_1' j_2')J' || S(1) || (j_1 j_2)J \right) \right.$$

$$+ (-1)^{j_1+j_2+J+1} \left((j_1' j_2')J' || S(1) || (j_2 j_1)J \right)$$

$$+ (-1)^{j_1'+j_2'+J'+1} \left((j_2' j_1')J' || S(1) || (j_1 j_2)J \right)$$

$$\left. + (-1)^{j_1+j_2+j_1'+j_2'+J+J'} \left((j_2' j_1')J' || S(1) || (j_2 j_1)J \right) \right\} \quad 2.101$$

where

$$\left((j_1' j_2')J || S(1) || (j_1 j_2)J \right)$$

$$= (-1)^{j_2+1+j_1'+J} \hat{j}_1' \hat{J} \begin{Bmatrix} j_1 & j_1' & 1 \\ J' & J & j_2 \end{Bmatrix} \delta_{j_2 j_2'} \langle j_1' || S || j_1 \rangle \quad 2.102$$

and

$$\langle (\ell', \tfrac{1}{2})j' || S || (\ell, \tfrac{1}{2})j \rangle = (-1)^{\ell'+1+\frac{1}{2}+j'} \sqrt{2}\, \hat{j}_1 \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & 1 \\ j' & j & \ell \end{Bmatrix} \langle \tfrac{1}{2} || S || \tfrac{1}{2} \rangle \delta_{\ell \ell'} \quad . \quad 2.103$$

Here we have

$$\langle \tfrac{1}{2} || S || \tfrac{1}{2} \rangle = \sqrt{\frac{3}{4}}$$

so that

$$\langle(\ell',\tfrac{1}{2})j'||S||(\ell,\tfrac{1}{2})j\rangle = (-1)^{\ell'+j'-\frac{1}{2}}\sqrt{\frac{3}{2}}\,\hat{j}_1\begin{Bmatrix}\frac{1}{2} & \frac{1}{2} & 1\\ j' & j & \ell\end{Bmatrix}\delta_{\ell',\ell} \qquad 2.104$$

Finally, we note that the BE2's may be expressed in terms of reduced matrix elements by simply performing the sum in Equation 2.84.

$$BE(\lambda;i\to f) = \frac{1}{2I_i+1}\sum_{m,m_f}|\langle f|M^{\mu}(\lambda)|i\rangle|^2$$

$$= \frac{1}{2I_i+1}\sum_{m,m_f}c^2(I_i,\lambda,I_f;M_i,\mu,M_f)|\langle I_f||M(\lambda)||I_i\rangle|^2$$

$$= \frac{2I_f+1}{2I_i+1}|\langle I_f||M(\lambda)||I_i\rangle|^2 \qquad 2.105$$

The other physical quantities of interest are the electric quadrupole moment

$$Q(I) \equiv \sqrt{\frac{16\pi}{5}}\,\langle II|Q_{20}|II\rangle$$

$$= \sqrt{\frac{16\pi}{5}}\,c(I2I;I0I)\langle I||M(E2)||I\rangle$$

and the magnetic dipole moment

$$M(I) \equiv \sqrt{\frac{4\pi}{3}}\,c(II1;I0I)\langle I||M(M1)||I\rangle$$

Also measured experimentally are E2/M1 mixing ratios. This ratio is a measure of the relative proportion of electric quadrupole to

magnetic dipole strength for transitions between levels where both transitions are allowed. The convention we use is that given in Nuclear Data Tables (74), and in terms of our operators this is written as

$$\delta = \left\{ \frac{(2I'+1)3}{(2I+1)\ 5} \right\}^{\frac{1}{2}} \frac{mc^2 \Delta E}{\hbar^2 c^2} \frac{\langle I' || M(E2) || I \rangle}{\langle I' || M(M1) || I \rangle}$$

for the transition $I' \rightarrow I$.

The percent of E2 is then

$$\%E2 = \frac{\delta^2}{1+\delta^2}$$

and conversely

$$\%M1 = \frac{1}{1+\delta^2} \quad .$$

Diagonalization and Basis Truncation Procedure

Since in principle both the core and the single particle states span an infinite basis, some choices about which states to include in a calculation must be made. Various choices of earlier investigators were discussed in the introduction, where it was seen that in the most recent work (46) 3 phonons (N = 3) and single particle orbits $3p_{1/2}$, $3p_{3/2}$, $2f_{5/2}$, $2f_{7/2}$, $1h_{9/2}$, and $1h_{13/2}$ were include in an N = 84 calculation. In Table IV we show the number of basis states for 2 neutrons in these single particle orbitals as N increases. The number of states rapidly becomes unmanageable, as we can only diagonalize matrices of order $\lesssim$200 reasonably fast on the IBM 360-65, 370-55 system at Iowa State.

Vanden Berghe's solution to this problem (46) was to treat only two-particle states with unperturbed energy up to 2.8 MeV. That is

$$E_1 + E_2 \leq 2.8 \text{ MeV}$$

How he arrived at 2.8 MeV is not discussed in his paper, but it appears to be because of his choice for single particle energies, in which $E_{7/2} = 1.4$, so $(7/2)^2 = 2.8$. Since it was our feeling that the contribution from four phonon states might not be negligible, we wanted a procedure which was both more systematic and would allow us to treat $N = 4$ states of the core. We were also wanted to treat the core and valence cluster on equal ground. That is, if we kept 4 phonons then since $4\hbar\omega$ is the maximum energy of the core the valence space should be truncated at about $4\hbar\omega$ also. Since the residual interaction will tend to split the coupled single particle states, this truncation would be carried out after the states are diagonalized with respect to $H_{residual}$.

Our procedure then is to first diagonalize

$$H_{residual}$$

in the basis of $H_{particle}$

$$|(n_1\ell_1 j_1)(n_2\ell_2 j_2)JM\rangle \quad .$$

This gives us diagonalized single particle states

$$|E_J^\alpha, J\rangle = \sum_{j_1 j_2} \langle j_1 j_2 | \alpha J\rangle |(j_1 j_2)J\rangle \qquad 2.106$$

where E_J^α is the α^{th} state of spin J with energy E_J^α.

Then only states with

$$E_J^\alpha \leq N\hbar\omega$$

are retained in the calculation.

The coupled (core and single particle) states are now written

$$|E_J^\alpha, J; N\ell\nu R; IM\rangle = \sum_{m_1 m_2} c(JRI; m_1 m_2 M) |E_J^\alpha, JM_1\rangle |N\ell\nu RM_2\rangle \qquad 2.107$$

As shown in Table IV, even this was not sufficient to truncate the basis down to a manageable size space, so a final restriction

$$E_J^\alpha + E_{core} \leq E_{cutoff}$$

was imposed. Then E_{cutoff} was varied as the total Hamiltonian was diagonalized for spin zero using reasonable values of $\hbar\omega$ and ξ. When the change in energies for the first 10 zero states is less than 1%, we take that as a sufficiently large basis. This procedure does not guarantee a small enough space to work with, but it turns out that we were able to achieve convergence with matrices no larger than 150 x 150.

Once the basis is chosen we must diagonalize

$$H_{cp}$$

between the basis states of Equation 2.60. This will produce energies E_I^β and eigenfunctions

$$|E_I^\beta, IM\rangle = \sum_{\substack{\alpha, J \\ N, \ell, \nu, R}} \langle \alpha, J; N\ell\nu R | \beta I\rangle |\alpha J; N\ell\nu R; IM\rangle \qquad 2.108$$

Table IV. Basis size for N = 84 space with three and four phonons for various values of E_{sp} and E_{cut}

N	E_{sp}(cutoff)	E_{tot}(cutoff)	Total # states	Number of spin zeros	Number of spin 1	Number of spin 4
3	2.6	-	280	10	23	43
3	3.6	-	836	28	57	124
3	4.4	-	1154	41	84	174
4	4.8	-	4540	110	253	578
4	4.8	6.0	589	24	41	93
4	4.8	6.5	665	26	48	105
4	4.8	6.6	677	26	48	108
4	4.8	6.7	958	35	65	148
4	4.8	6.8	966	37	65	150
4	4.8	7.0	1448	46	94	210

which may of course be written in terms of the original basis

$$|E_I^{\beta}, IM\rangle = \sum_{\substack{\alpha J \\ j_1 j_2 \\ N\ell\nu R}} \langle \alpha J; N\ell\nu R|\beta I\rangle\langle j_1 j_2|\alpha J\rangle|(j_1 j_2)J, N\ell\nu R; IM\rangle \qquad 2.109$$

Then any operator we wish to consider has reduced matrix elements of the form

$$\langle E_{I'}^{\beta'}||\theta||E_I^{\beta}\rangle = \sum_{\substack{\alpha' J' \\ j_1' j_2' \\ N'\ell'\nu' R'}} \sum_{\substack{\alpha J \\ j_1 j_2 \\ N\ell\nu R}}$$

$$\times \langle \alpha J; N\ell\nu R||\beta I\rangle\langle j_1 j_2||J\rangle\langle \alpha' J'; N'\ell'\nu' R'\rangle\langle I'\rangle\langle j_1' j_2'|\alpha' I'\rangle$$

$$\times \langle (j_1' j_2')J'; N'\ell'\nu' R', I'||\theta||(j_1 j_2)J; N\ell\nu R; I\rangle \qquad 2.110$$

which may be easily evaluated.

CHAPTER 3. RESULTS AND DISCUSSION

Nuclei With N = 84. Pairing Residual Interaction

Basis selection

Following the procedure discussed in Chapter 2, last section, we first diagonalized the particle part of the Hamiltonian using the coupled single particle states as a basis. The pairing strength was taken as $G = 27/A$ with $A = 144$, and single particle energies used were $E_{1/2} = 1.520$, $E_{3/2} = 1.205$, $E_{5/2} = 2.020$, $E_{9/2} = 1.40$, and $E_{13/2} = 2.60$ (MeV). These single particle energies were determined in a calculation for the N = 83 nuclei (35, 46). The matrix of the entire Hamiltonian for states of total spin I = 0 was then generated and diagonalized. The basis used included up to four phonon states with $\hbar\omega = 1.2$ MeV and $\xi = 2.0$. In the calculation E_{cut} was varied and the diagonalized two-particle states with energies less than $4\hbar\omega$ were retained. The levels were seen to converge by $E_{cut} = 6.8$ MeV; i.e., increasing E_{cut} above 6.8 MeV did not produce a change greater than 1% in these energies. This convergence is shown in Figure 2, where the energies of some 0^+ states are shown as a function of E_{cut}. The number of states of spin zero and spin four (the largest matrix of a given spin in our basis) are also indicated.

Fits to experimental spectra

The proper (converged) basis space having been determined, we generated the matrix elements of H_{cp} for each spin in this basis. Then we used a chi-squared minimization routine (stepit) to determine

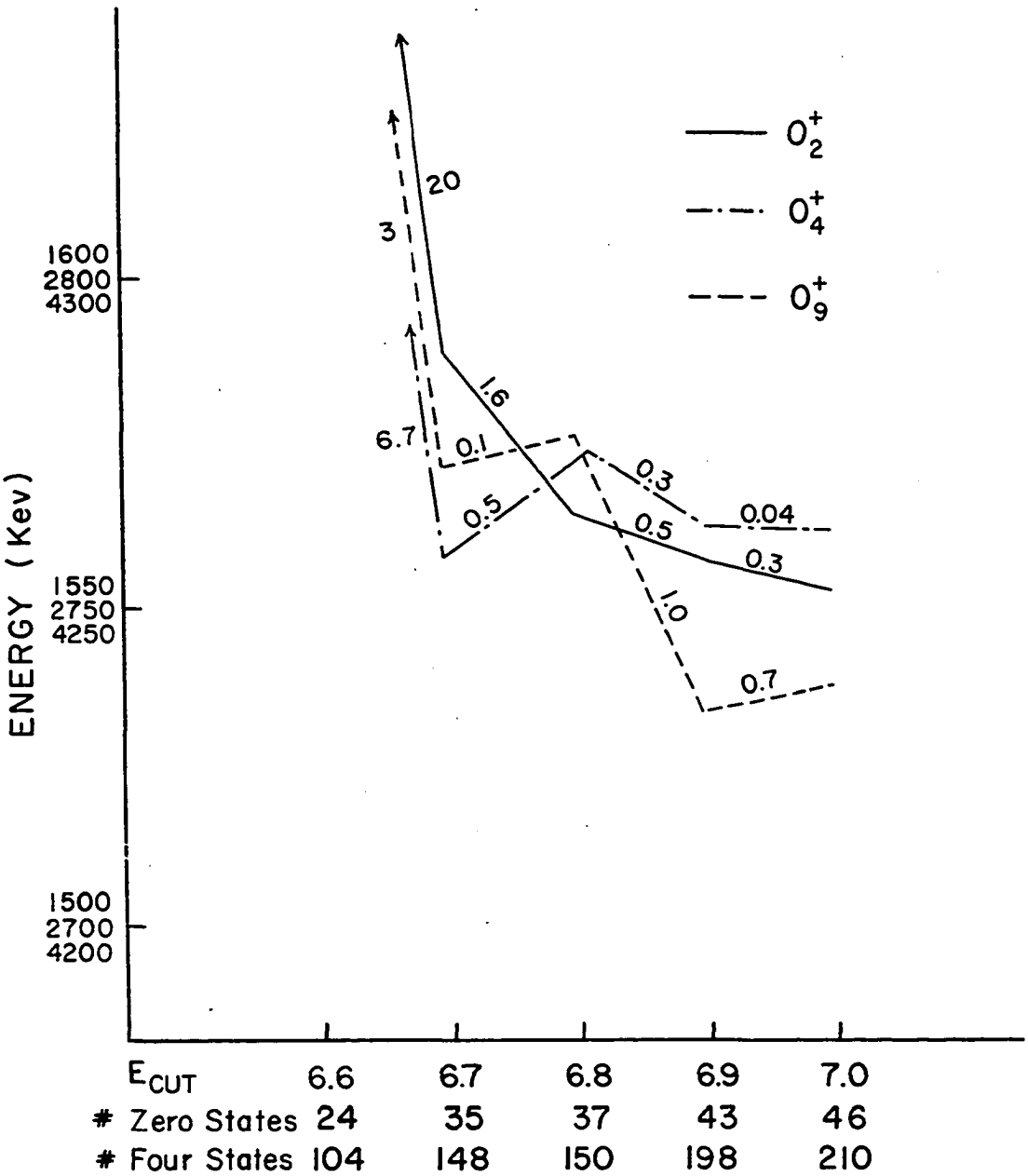

Figure 2. N = 84 energy level convergence for a residual pairing interaction. The behavior of the second, fourth, and ninth spin zero levels is followed as a function of increasing E_{cut} and basis size. The percent change between points is indicated.

the best values of the parameters $\hbar\omega$ and ξ for the lowest few positive parity states in each of the N = 84 nuclei: ^{140}Ba, ^{142}Ce, ^{144}Nd, ^{146}Sm, and ^{148}Gd. The systematics of the energies of the low-lying states for the N = 84 nuclei are shown in Figure 3. The value of G was fixed at 27/A MeV for A = 144, and was not changed as A changed since the single particle states had already been diagonalized and we did not want to generate a new set of core-particle matrix elements for each nucleus. This model then has only two parameters to vary. The fitted values for $\hbar\omega$ and ξ as a function of increasing A (and Z) are shown in Figure 4. The values of $\hbar\omega$ are seen to increase linearly as pairs of protons are added to the core, showing they tend to increase the stiffness of the core to quadrupole vibrations. This is fairly easily understood. From a hydrodynamic point of view, the stiffness constant depends in part upon the energy difference between a quadrupole and spherical shape. The addition of more protons tends to favor more a spherical shape and this energy difference rises and along with it the stiffness constant rises. Similarly the core-particle coupling strength is seen to decrease linearly as a function of Z. The explanation for this can be understood with the same reasoning as for the stiffness. As protons are added to the core and a more spherical shape is favored, the interaction between the core and particles, which is an interaction with the surface shape deformation, decreases. The experimental references are: ^{140}Ba (47); ^{142}Ce (75); ^{144}Nd (76); ^{146}Sm (77); and ^{148}Gd (78). The behavior of the low-lying states as ξ is varied for a fixed value of $\hbar\omega$ is shown in Figure 5.

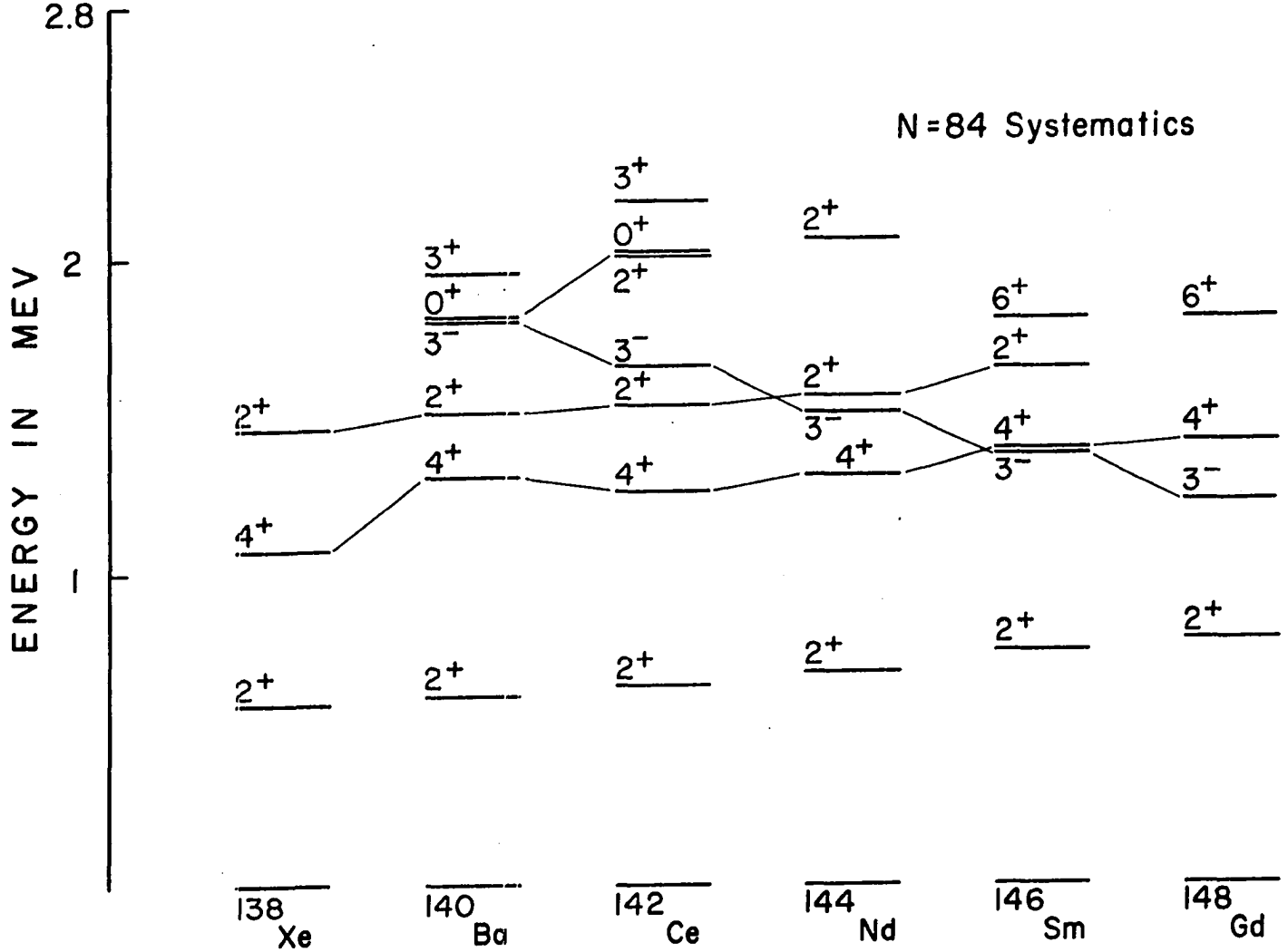

Figure 3. Systematics of the experimental values of the lowest energy states for the N = 84 nuclei.

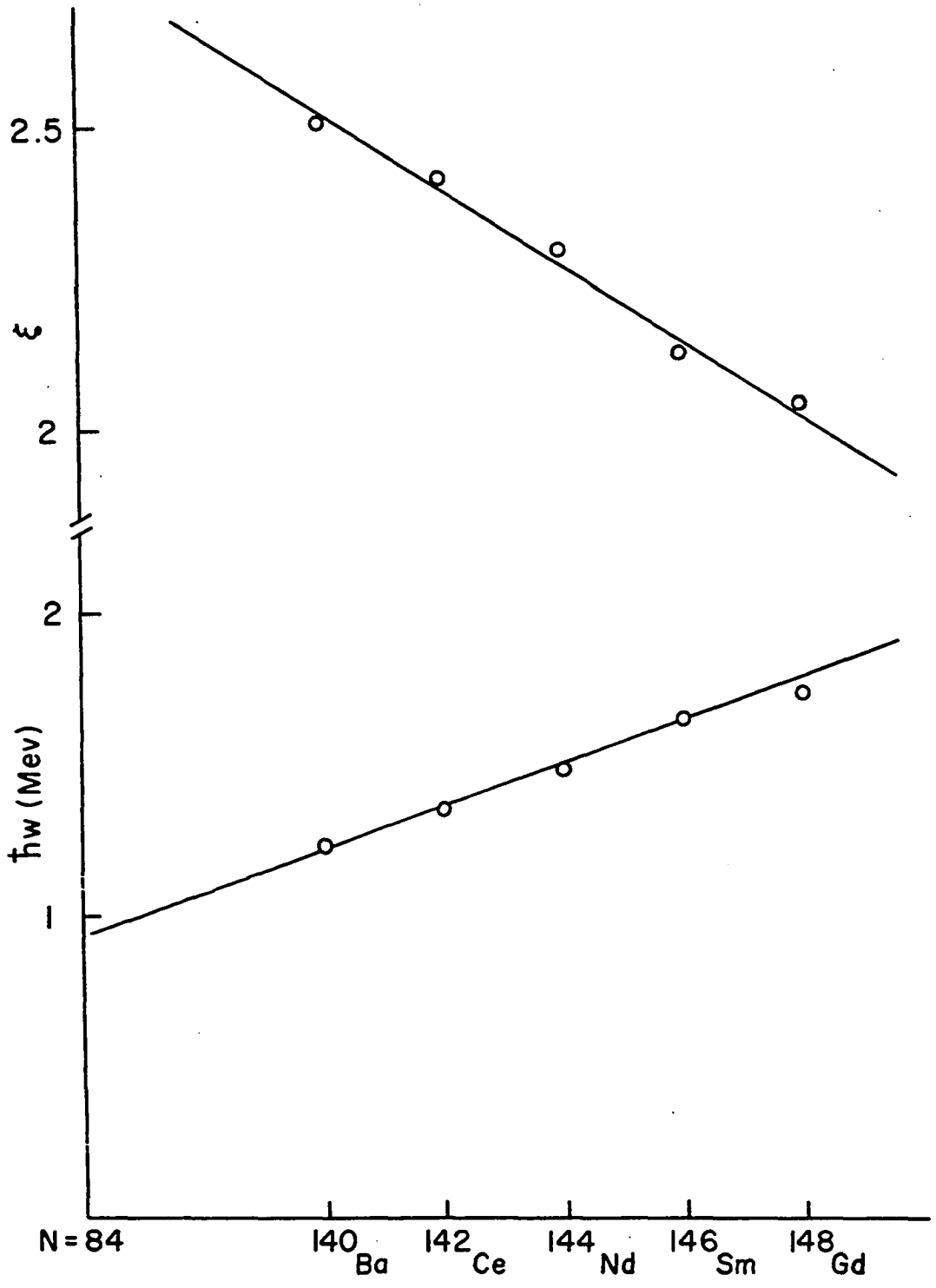

Figure 4. Model parameters $\hbar\omega$ and ξ as a function of A (and N) for the N = 84 nuclei. These were determined by a chi-squared fit to the experimental energies.

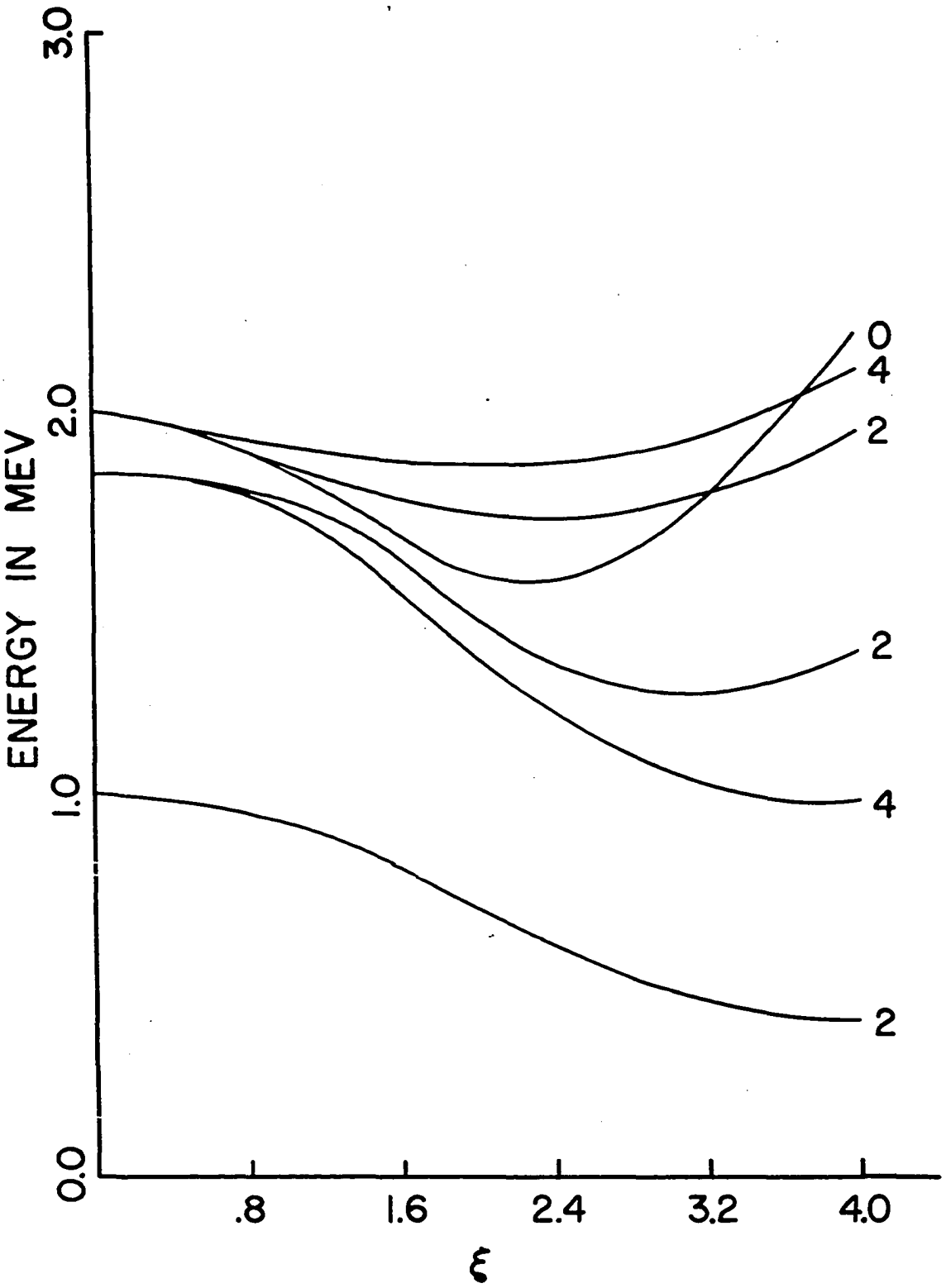

Figure 5. Behavior of the model states of lowest energy for the N = 84 nuclei as a function of increasing ξ for $\hbar\omega$ = 1.0 MeV.

The energy spectra which we found vs. the experimental spectra are displayed in Figures 6 to 10 for all of the N = 84 nuclei studied. The theoretical energies for states below 3 MeV are shown in Table V. In Figure 11 we also compare our results and those of Vanden Berghe (46) for ^{140}Ba. Since he did not determine a best fit for each nucleus individually our better agreement is not surprising, but we will see later that we find a significant contribution from states excluded in his treatment and this influences the electromagnetic properties as well as the energy levels.

The agreement between the theoretical and experimental energies is seen to be excellent for the low-lying states. For each of the experimentally identified positive parity states, a corresponding state in the theoretical spectra can easily be identified, although only ^{140}Ba and ^{142}Ce have a very complete experimental energy spectrum. The model eigenfunctions for each state were found to be a mixture of quite a large number of basis states. Table VI shows the composition for the lower-lying states of each spin for ^{140}Ba. Note that the 0_2^+ state has significant four phonon contribution; these states were excluded in Vanden Berghe's treatment.

Electromagnetic properties

The calculated B(E2) and B(M1) values and reduced matrix elements for each nucleus are shown in Tables VII and VIII. We used an effective neutron charge of e_n^{eff} = .2 e to calculate the B(E2) values which we took because it gave best agreement with the experimental B(E2, $2_1^+ \rightarrow 0_1^+$). This value is considerably smaller than the values of e_n^{eff} = e and

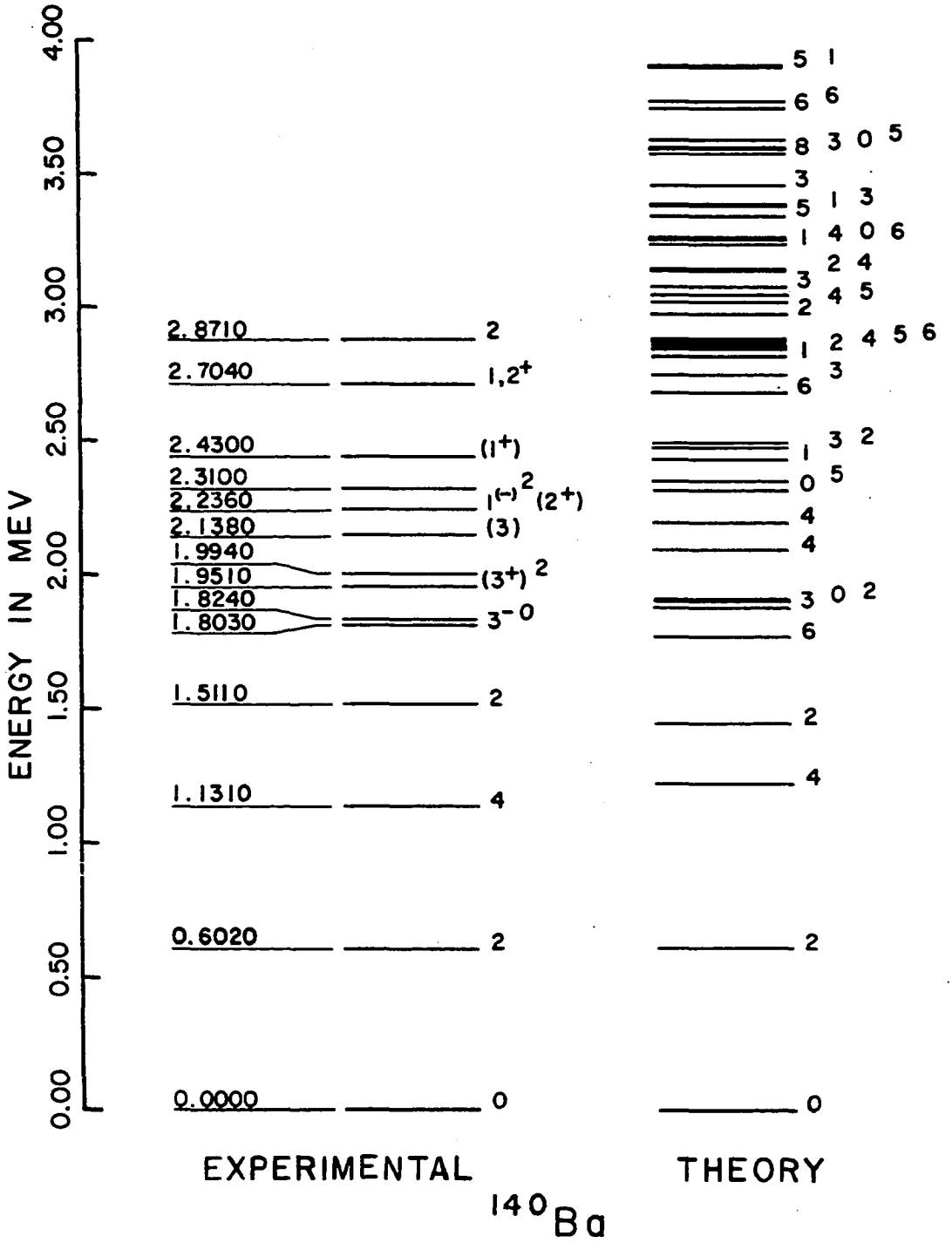

Figure 6. Theoretical and experimental levels of ^{140}Ba. The theoretical levels were computed using a residual pairing interaction. Only those positive parity theoretical levels are shown for which $I \leq 6$. The model parameters were $\hbar\omega = 1.227$ MeV, $\xi = 2.51$.

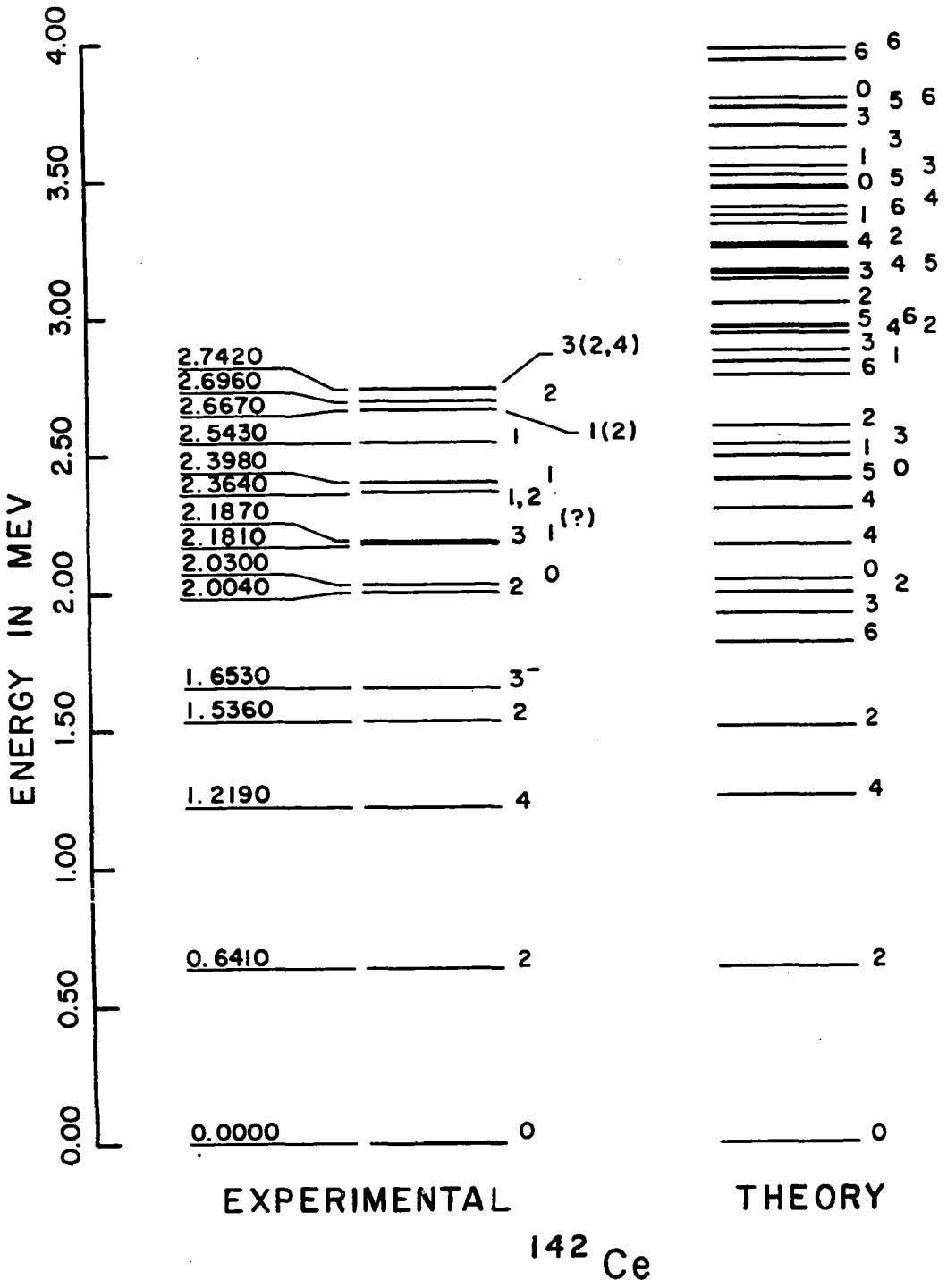

Figure 7. Theoretical and experimental levels of ^{142}Ce. The theoretical levels were computed using a residual pairing interaction. Only those positive parity theoretical levels are shown for which $I \leq 6$. The model parameters were $\hbar\omega = 1.348$ MeV, $\xi = 2.42$.

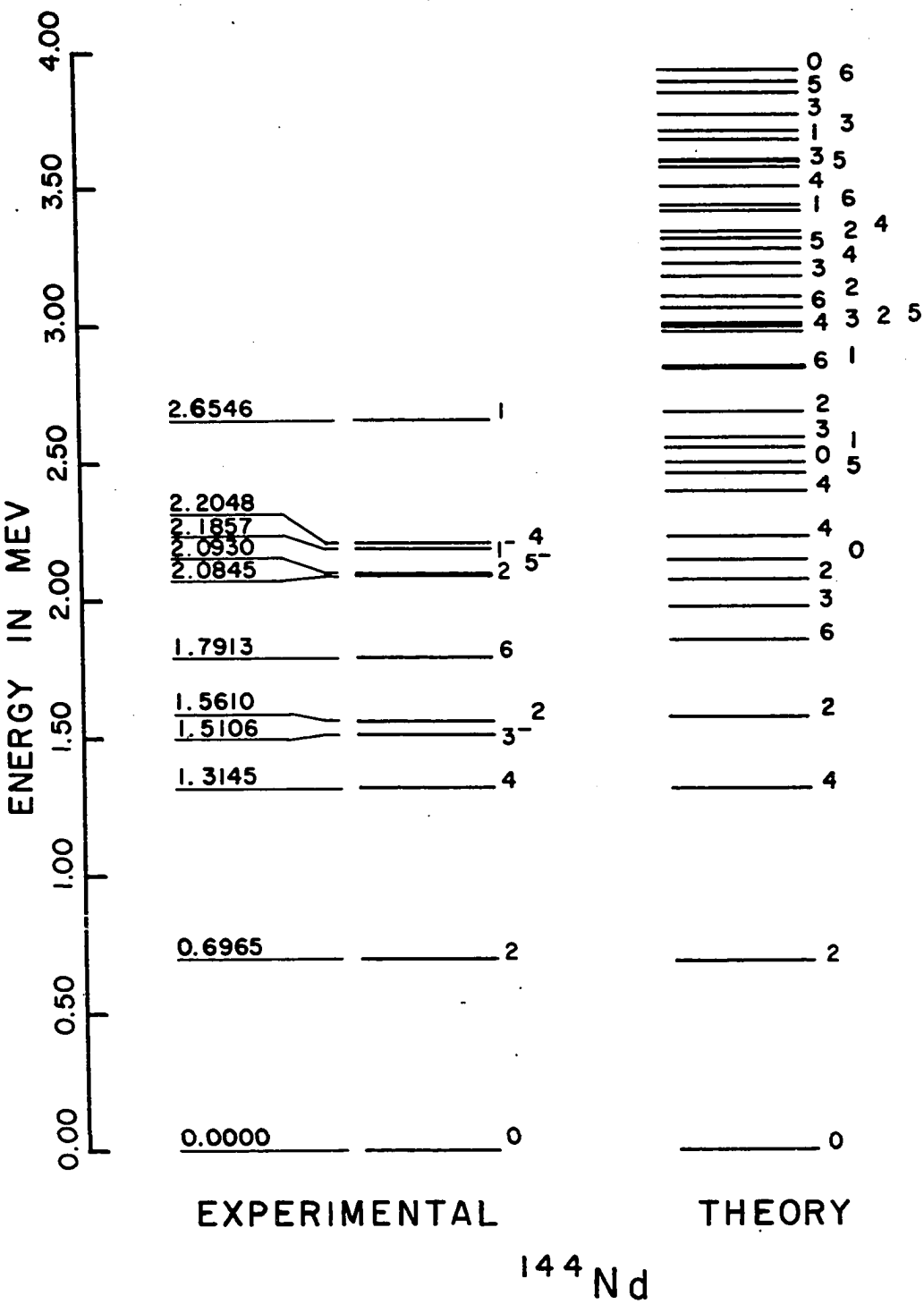

Figure 8. Theoretical and experimental levels of ^{144}Nd. The theoretical levels were computed using a residual pairing interaction. Only those positive parity theoretical levels are shown for which $I \leq 6$. The model parameters were $\hbar\omega = 1.449$ MeV, $\xi = 2.29$.

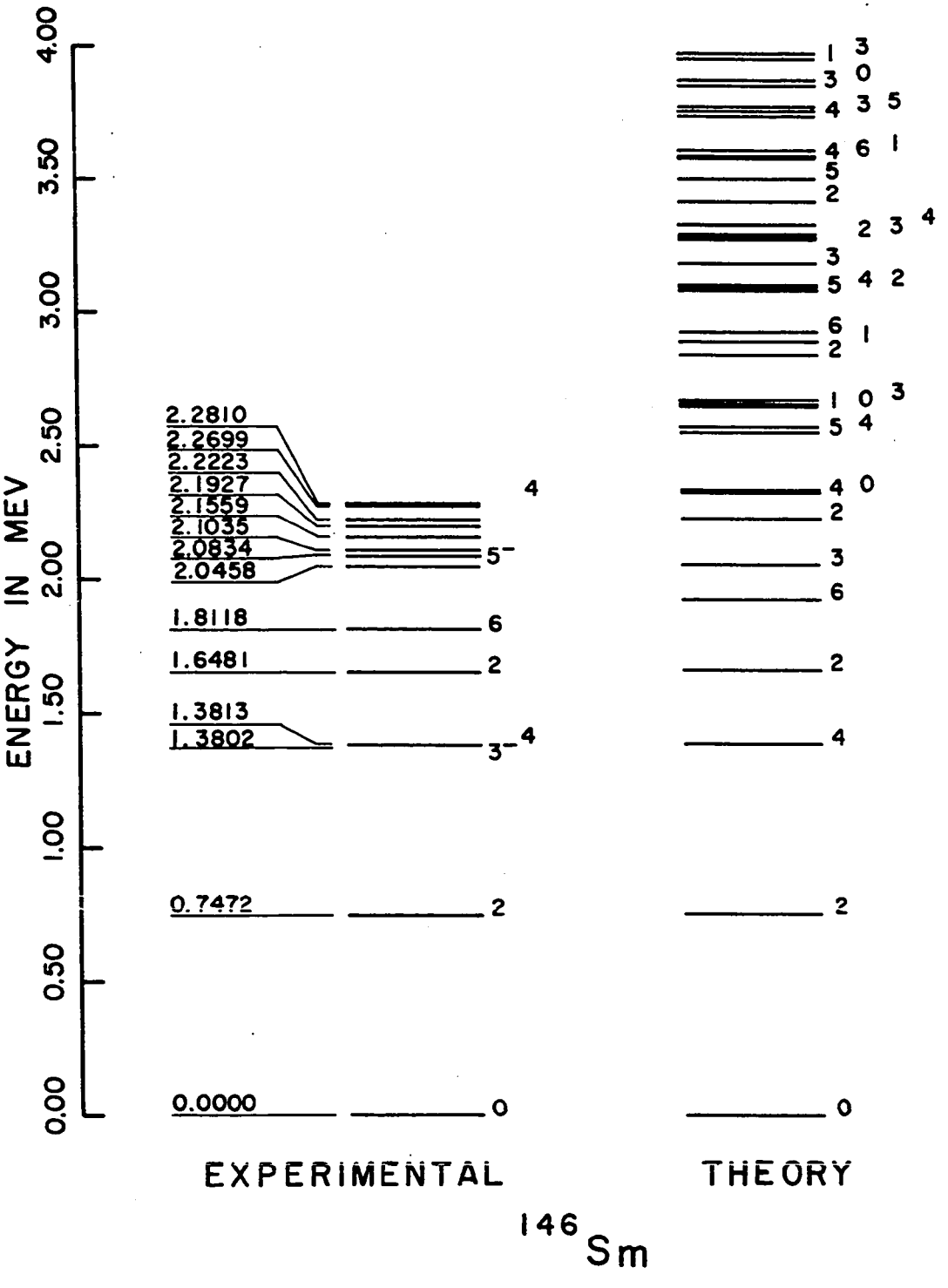

Figure 9. Theoretical and experimental levels of ^{146}Sm. The theoretical levels were computed using a residual pairing interaction. Only those positive parity theoretical levels are shown for which $I \leq 6$. The model parameters were $\hbar\omega = 1.650$ MeV, $\xi = 2.13$.

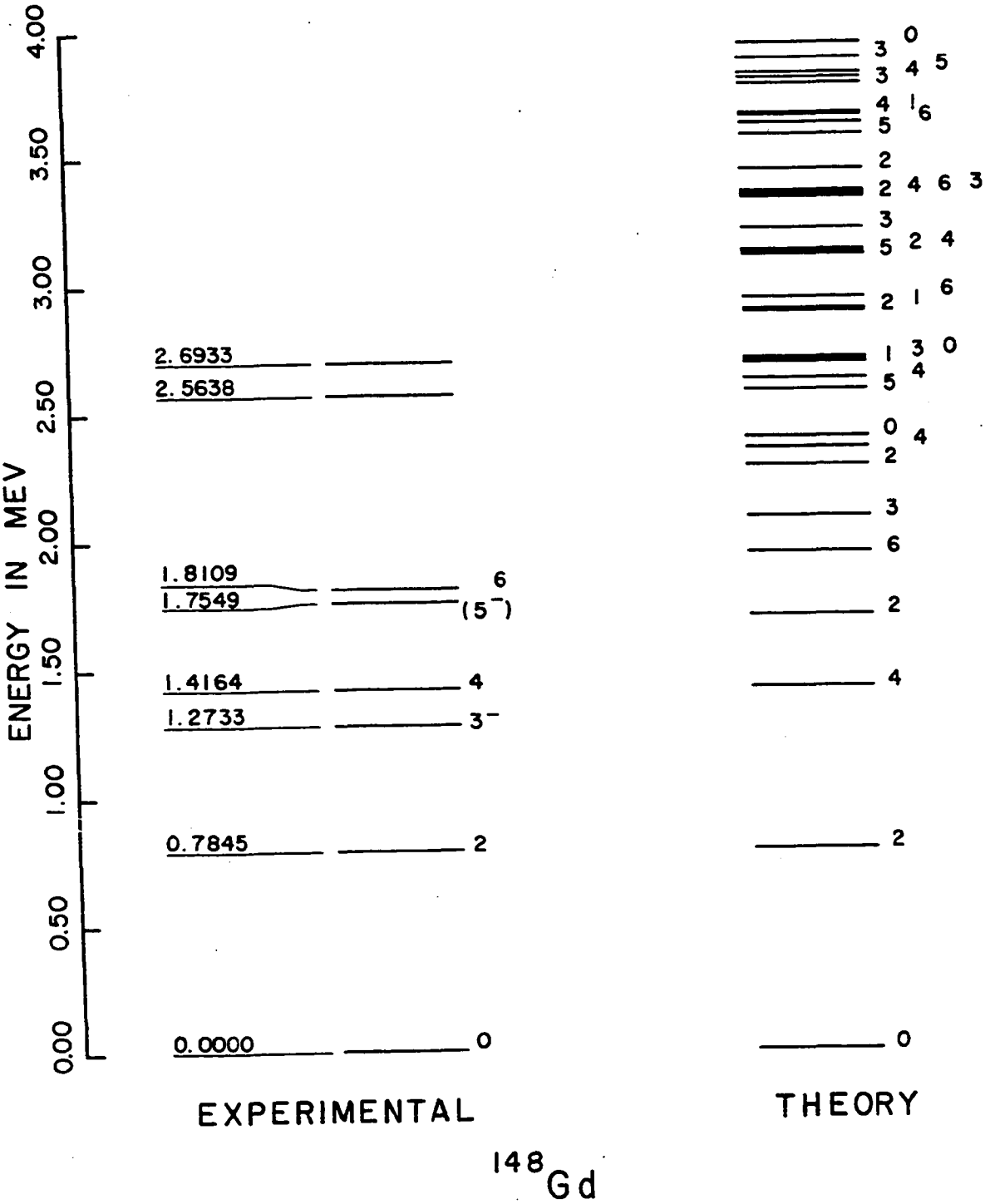

Figure 10. Theoretical and experimental levels of ^{148}Gd. The theoretical levels were computed using a residual pairing interaction. Only those positive parity theoretical levels are shown for which $I \leq 6$. The model parameters were $\hbar\omega = 1.726$ MeV, $\xi = 2.05$.

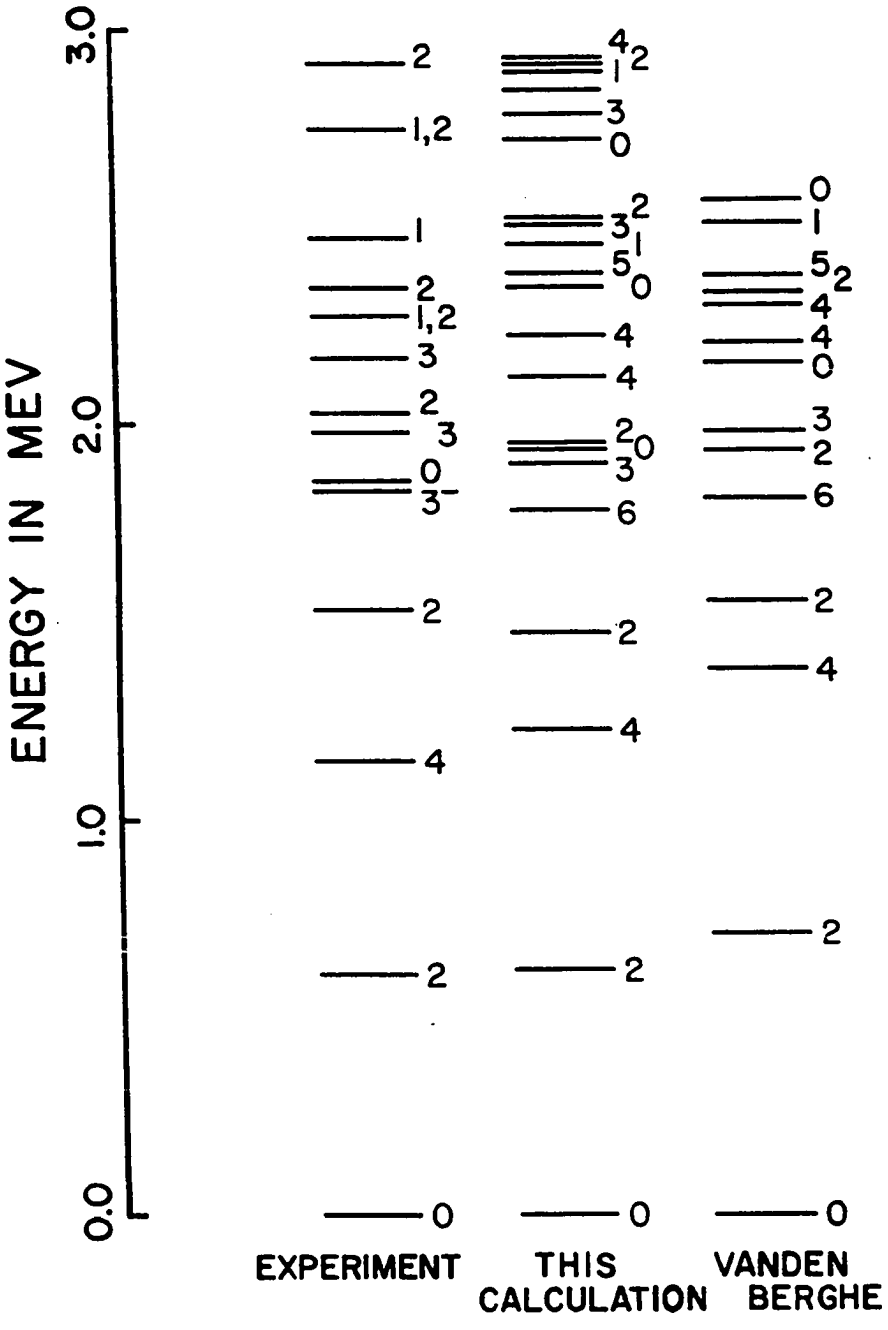

Figure 11. Comparison between energy spectra predicted for ^{140}Ba in this work and the average N = 84 spectra computed by Vanden Berghe (46).

Table V. The theroretical energies of the positive parity states for the N = 84 nuclei. Only energies less than 3 MeV are shown

State	^{140}Ba	^{142}Ce	^{144}Nd	^{146}Sm	^{148}Gd
2_1	.605	.638	.684	.750	.785
4_1	1.216	1.259	1.311	1.383	1.417
2_2	1.442	1.512	1.570	1.661	1.693
6_1	1.759	1.813	1.849	1.920	1.939
3_1	1.871	1.922	1.971	2.050	2.082
0_2	1.896	2.043	2.138	2.329	2.390
2_3	1.900	1.996	2.069	2.223	2.378
4_2	2.085	2.168	2.225	2.320	2.352
4_3	2.186	2.298	2.388	2.565	2.620
0_3	2.304	2.412	2.493	2.650	2.710
5_1	2.338	2.407	2.454	2.545	2.573
1_1	2.419	2.489	2.549	2.644	2.679
3_2	2.466	2.533	2.586	2.666	2.694
2_4	2.482	2.600	2.682	2.836	2.884
6_2	2.669	2.786	2.839	2.918	2.935
3_3	2.737	2.875	2.988	3.174	3.206
1_2	2.803	2.835	2.847	2.884	2.891

Table V. (Continued)

State	$^{140}_{Ba}$	$^{142}_{Ce}$	$^{144}_{Nd}$	$^{146}_{Sm}$	$^{148}_{Gd}$
2_5	2.834	2.938	2.993	3.092	3.113
4_4	2.848	2.937	2.975	3.091	3.122
5_2	2.866	2.954	3.001	3.082	3.101
6_3	2.871	2.967	3.055	3.265	3.338
2_6	2.963	3.046	3.102	3.270	3.325

Table VI. Eigenvectors for ^{140}Ba, pairing residual interaction

#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
1	0									
		-.2639D 00	3	3	0	0	0	0	0	0
		-.1148D 00	3	3	0	2	0	0	0	0
		-.1651D 00	5	5	0	0	0	0	0	0
		.1141D 00	3	3	2	1	1	0	2	0
		-.6072D 00	7	7	0	0	0	0	0	0
		-.2146D 00	7	7	0	2	0	0	0	0
		.1199D 00	3	7	4	2	2	0	4	0
		-.2011D 00	9	9	0	0	0	0	0	0
		-.3369D 00	3	7	2	1	1	0	2	0
		-.1669D 00	3	7	2	2	2	0	2	0
		-.1068D 00	3	7	2	3	1	0	2	0
		-.1951D 00	13	13	0	0	0	0	0	0
		.3516D 00	7	7	2	1	1	0	2	0
		.1011D 00	7	7	2	3	1	0	2	0
		-.1550D 00	7	7	4	2	2	0	4	0
TOTAL=		.9999999999999998D 00								

#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
2	0									
		.2033D 00	3	3	0	2	0	0	0	0
		-.1610D 00	5	5	0	0	0	0	0	0
		-.1123D 00	3	3	2	1	1	0	2	0
		-.4527D 00	7	7	0	0	0	0	0	0
		.4190D 00	7	7	0	2	0	0	0	0
		-.1854D 00	3	7	4	2	2	0	4	0
		-.1145D 00	3	7	4	3	3	0	4	0
		.1185D 00	7	7	0	3	3	1	0	0
		.1039D 00	7	7	0	4	0	0	0	0
		-.2551D 00	9	9	0	0	0	0	0	0
		.1917D 00	3	7	2	1	1	0	2	0
		.1919D 00	3	7	2	2	2	0	2	0
		.2626D 00	3	7	2	3	1	0	2	0
		-.1633D 00	13	13	0	0	0	0	0	0
		.1206D 00	13	13	0	2	0	0	0	0
		-.1197D 00	7	7	6	3	3	0	6	0
		-.1329D 00	7	7	2	2	2	0	2	0
		-.2459D 00	7	7	2	3	1	0	2	0
		.1840D 00	7	7	4	2	2	0	4	0
		.1071D 00	7	7	4	4	2	0	4	0
TOTAL=		.9999999999999996D 00								

Table VI. (Continued)

#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
1	1									
		0.1738D 00	1	3	1	0	0	0	0	1
		0.2333D 00	1	7	3	1	1	0	2	1
		0.1389D 00	1	7	3	2	2	0	2	1
		-.1225D 00	3	7	5	2	2	0	4	1
		0.3215D 00	5	7	1	0	0	0	0	1
		-.1444D 00	3	3	2	1	1	0	2	1
		0.1208D 00	5	7	1	2	0	0	0	1
		0.1820D 00	7	9	1	0	0	0	0	1
		0.1435D 00	3	7	3	1	1	0	2	1
		0.1207D 00	3	7	3	2	2	0	2	1
		-.2629D 00	3	7	4	2	2	0	4	1
		-.1331D 00	3	7	4	3	3	0	4	1
		0.1594D 00	3	7	2	1	1	0	2	1
		0.1146D 00	3	7	2	2	2	0	2	1
		-.1307D 00	5	7	3	1	1	0	2	1
		-.2513D 00	5	7	2	1	1	0	2	1
		-.2150D 00	7	7	6	3	3	0	6	1
		-.3973D 00	7	7	2	1	1	0	2	1
		-.1080D 00	7	7	2	3	1	0	2	1
		-.2114D 00	7	9	2	1	1	0	2	1
		0.3716D 00	7	7	4	2	2	0	4	1
TOTAL=		0.1000000300000D 01								

#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
2	1									
		0.1699D 00	1	7	3	1	1	0	2	1
		0.2567D 00	5	7	1	0	0	0	0	1
		-.2929D 00	5	7	1	1	1	0	2	1
		-.1107D 00	5	7	1	2	2	0	2	1
		-.5893D 00	7	9	1	0	0	0	0	1
		0.2718D 00	7	9	1	1	1	0	2	1
		-.1584D 00	7	9	1	2	0	0	0	1
		-.1435D 00	3	9	3	1	1	0	2	1
		-.1828D 00	5	7	3	1	1	0	2	1
		0.3551D 00	7	9	3	1	1	0	2	1
		-.1332D 00	7	9	3	2	2	0	4	1
		-.1450D 00	7	9	5	2	2	0	4	1
		0.2352D 00	7	9	2	1	1	0	2	1
TOTAL=		0.1000000000000D 01								

Table VI. (Continued)

#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
1	2									
		0.2306D 00	3	3	0	1	1	0	2	2
		0.1193D 00	5	5	0	1	1	0	2	2
		0.5075D 00	7	7	0	1	1	0	2	2
		-.1658D 00	3	7	4	1	1	0	2	2
		0.1197D 00	7	7	0	2	2	0	2	2
		0.1409D 00	7	7	0	3	1	0	2	2
		0.2454D 00	3	7	2	0	0	0	0	2
		0.1324D 00	9	9	0	1	1	0	2	2
		-.2265D 00	3	7	2	1	1	0	2	2
		0.1428D 00	3	7	2	2	0	0	0	2
		-.1391D 00	3	7	2	2	2	0	2	2
		0.2054D 00	3	7	2	2	2	0	4	2
		0.1491D 00	13	13	0	1	1	0	2	2
		-.1033D 00	7	7	6	2	2	0	4	2
		-.2851D 00	7	7	2	0	0	0	0	2
		-.1472D 00	7	7	2	2	0	0	0	2
		0.1537D 00	7	7	2	2	2	0	2	2
		-.2134D 00	7	7	2	2	2	0	4	2
		0.2007D 00	7	7	4	1	1	0	2	2
TOTAL=		0.1000000000000002D 01								

#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
2	2									
		0.1348D 00	3	3	0	2	2	0	2	2
		0.1983D 00	3	3	2	1	1	0	2	2
		-.1612D 00	7	7	0	1	1	0	2	2
		-.2249D 00	3	7	3	1	1	0	2	2
		0.3825D 00	7	7	0	2	2	0	2	2
		0.1237D 00	3	7	4	2	2	0	4	2
		0.2327D 00	3	7	2	0	0	0	0	2
		0.1286D 00	3	7	2	2	0	0	0	2
		0.1091D 00	3	7	2	2	2	0	2	2
		0.1264D 00	3	7	2	3	3	0	3	2
		0.3345D 00	7	7	2	0	0	0	0	2
		0.4652D 00	7	7	2	1	1	0	2	2
		0.1276D 00	7	7	2	2	0	0	0	2
		0.1537D 00	7	7	2	3	1	0	2	2
		-.1175D 00	7	7	2	3	3	0	3	2
		-.1066D 00	7	7	2	3	3	0	4	2
		-.1045D 00	7	7	4	1	1	0	2	2
		-.2494D 00	7	7	4	2	2	0	4	2
TOTAL=		0.1000000000000002D 01								

Table VI. (Continued)

#	SPIN	COFF.	J1	J2	J	N	L	NU	R	I
3	2									
		-.1254D 00	3	3	0	1	1	0	2	2
		0.1110D 00	3	3	0	2	2	0	2	2
		-.1504D 00	3	3	2	0	0	0	0	2
		-.1044D 00	5	5	0	1	1	0	2	2
		-.3624D 00	7	7	0	1	1	0	2	2
		0.2236D 00	3	7	3	1	1	0	2	2
		-.2265D 00	3	7	4	1	1	0	2	2
		0.1960D 00	7	7	0	2	2	0	2	2
		0.1137D 00	3	7	3	2	2	0	2	2
		-.1303D 00	3	7	4	2	2	0	2	2
		-.1379D 00	9	9	0	1	1	0	2	2
		-.2278D 00	3	7	2	1	1	0	2	2
		-.1138D 00	3	7	2	2	2	0	4	2
		0.1085D 00	3	7	2	3	3	0	3	2
		-.1187D 00	13	13	0	1	1	0	2	2
		-.1440D 00	5	7	2	0	0	0	0	2
		-.1984D 00	7	7	6	2	2	0	4	2
		-.1040D 00	7	7	6	3	3	0	4	2
		-.3446D 00	7	7	2	0	0	0	0	2
		-.1169D 00	7	7	2	2	0	0	0	2
		-.1753D 00	7	7	2	2	2	0	2	2
		0.1650D 00	7	7	2	2	2	0	4	2
		0.2784D 00	7	7	4	1	1	0	2	2
		0.1406D 00	7	7	4	2	2	0	2	2
TOTAL=		0.1000000000000002D 01								
1	3									
		-.1281D 00	1	7	4	1	1	0	2	3
		0.1559D 00	1	7	3	0	0	0	0	3
		-.1796D 00	3	7	5	1	1	0	2	3
		-.1043D 00	3	7	5	2	2	0	2	3
		0.1203D 00	3	3	2	1	1	0	2	3
		0.4100D 00	3	7	3	0	0	0	0	3
		-.1972D 00	3	7	4	1	1	0	2	3
		0.1897D 00	3	7	3	2	0	0	0	3
		-.1569D 00	3	7	3	2	2	0	4	3
		0.1334D 00	7	7	0	3	3	0	3	3
		0.2142D 00	3	7	2	1	1	0	2	3
		0.1176D 00	5	7	2	1	1	0	2	3

Table VI. (Continued)

		-.2263D 00	7	7	6	2	2	4	4	3
		.3125D 00	7	7	2	1	1	2	2	3
		.2769D 00	7	7	2	2	2	0	2	3
		.1457D 00	7	7	2	2	2	2	4	3
		.3689D 00	7	7	4	1	1	2	2	3
		-.1153D 00	7	7	4	2	2	4	4	3
		.1142D 00	7	7	4	3	1	2	2	3
TOTAL=		.1[illegible]2D 01								
*	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
1	4									
		.1589D 00	3	3	1	2	2	0	4	4
		-.1272D 00	3	3	2	1	1	2	2	4
		-.2611D 00	3	7	4	0	0	0	0	4
		-.1174D 00	3	7	3	1	1	0	2	4
		.1841D 00	3	7	4	1	1	2	2	4
		-.1374D 00	3	7	4	2	0	2	0	4
		.3412D 00	7	7	1	2	2	0	4	4
		.2751D 00	3	7	2	1	1	0	2	4
		.1146D 00	3	7	2	2	2	1	2	4
		-.1887D 00	3	7	2	2	2	0	4	4
		.1769D 00	3	7	2	2	1	1	2	4
		.1212D 00	3	7	2	2	2	1	0	4
		-.2626D 00	7	7	6	1	1	0	2	4
		-.1176D 00	7	7	6	2	2	2	2	4
		-.3196D 00	7	7	2	1	1	1	2	4
		-.1183D 00	7	7	2	3	1	1	2	4
		-.1234D 00	7	7	2	2	2	0	0	4
		.3404D 00	7	7	4	1	1	1	0	4
		-.1292D 00	7	7	4	1	1	2	2	4
		.1561D 00	7	7	4	2	0	1	0	4
		-.1287D 00	7	7	4	2	2	0	2	4
TOTAL=		.1[illegible]2D 01								

Table VI. (Continued)

#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
2	4									
		0.2005D 00	1	7	4	0	0	0	0	4
		0.1264D 00	1	7	3	1	1	0	2	4
		0.1214D 00	3	7	5	1	1	0	2	4
		0.1561D 00	3	3	0	2	2	0	4	4
		-.1004D 00	3	7	5	2	2	0	4	4
		0.1907D 00	3	7	4	0	0	0	0	4
		0.2629D 00	3	7	3	1	1	0	2	4
		0.3584D 00	7	7	0	2	2	0	4	4
		0.1831D 00	3	7	2	1	1	0	2	4
		0.1050D 00	9	9	0	2	2	0	4	4
		0.1435D 00	3	7	2	3	3	0	6	4
		0.1091D 00	13	13	0	2	2	0	4	4
		-.1319D 00	5	7	4	0	0	0	0	4
		0.1403D 00	5	7	2	1	1	0	2	4
		0.3051D 00	7	7	6	1	1	0	2	4
		0.1375D 00	7	7	6	2	2	0	2	4
		0.1541D 00	7	7	2	2	2	0	4	4
		0.1288D 00	7	7	2	3	3	0	4	4
		-.1314D 00	7	7	2	3	3	0	6	4
		-.3809D 00	7	7	4	0	0	0	0	4
		0.1075D 00	7	7	4	2	2	0	4	4
TOTAL=		0.1000000000000122D 01								

#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
1	5									
		0.1142D 00	1	7	4	1	1	0	2	5
		-.4760D 00	3	7	5	0	0	0	0	5
		0.1508D 00	1	7	3	1	1	0	2	5
		0.3388D 00	3	7	5	1	1	0	2	5
		-.1952D 00	3	7	5	2	0	0	0	5
		0.2316D 00	3	7	3	1	1	0	2	5
		0.1128D 00	3	7	4	1	1	0	2	5
		0.1193D 00	3	7	3	2	2	0	2	5
		0.1163D 00	3	7	4	2	2	0	2	5
		-.1034D 00	3	7	4	2	2	0	4	5
		-.4034D 00	7	7	6	1	1	0	2	5
		0.2588D 00	7	7	6	2	2	0	4	5
		0.1481D 00	7	7	2	2	2	0	4	5
		-.1969D 00	7	7	4	1	1	0	2	5
		-.1584D 00	7	7	4	2	2	0	2	5
		0.1332D 00	7	7	4	2	2	0	4	5
TOTAL=		0.1000000000000001D 01								

Table VI. (Continued)

#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
2	5									
		-.2122D 00	3	7	5	0	0	0	0	5
		0.1482D 00	1	7	3	1	1	0	2	5
		0.2457D 00	3	7	5	1	1	0	2	5
		-.1170D 00	1	7	3	2	2	0	4	5
		-.1488D 00	3	7	5	2	2	0	4	5
		-.2268D 00	3	7	4	1	1	0	2	5
		0.1049D 00	3	7	4	2	2	0	2	5
		-.1576D 00	3	7	3	2	2	0	4	5
		0.1086D 00	3	7	4	2	2	0	4	5
		0.2721D 00	5	7	5	0	0	0	0	5
		-.1532D 00	5	7	5	1	1	0	2	5
		0.2097D 00	7	9	7	1	1	0	2	5
		-.1712D 00	5	7	3	1	1	0	2	5
		-.1142D 00	7	7	6	1	1	0	2	5
		0.2908D 00	7	7	6	2	2	0	2	5
		0.1141D 00	7	7	6	3	3	0	3	5
		-.2468D 00	7	9	5	0	0	0	0	5
		0.1090D 00	7	9	3	1	1	0	2	5
		-.2221D 00	7	7	2	2	2	0	4	5
		0.3847D 00	7	7	4	1	1	0	2	5
		-.1055D 00	7	7	4	2	2	0	4	5
TOTAL=		0.1000000000000D 01								

#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
1	6									
		-.1447D 00	3	7	5	1	1	0	2	6
		0.1075D 00	3	3	2	2	2	0	4	6
		0.3343D 00	3	7	4	1	1	0	2	6
		0.1016D 00	5	7	6	1	1	0	2	6
		0.1327D 00	3	7	4	2	2	0	2	6
		0.1004D 00	3	7	3	2	2	0	4	6
		-.1187D 00	3	7	4	2	2	0	4	6
		0.1032D 00	3	7	4	3	1	0	2	6
		-.1171D 00	7	7	0	3	3	0	6	6
		-.1544D 00	3	7	2	2	2	0	4	6
		0.5669D 00	7	7	6	0	0	0	0	6
		-.3173D 00	7	7	6	1	1	0	2	6
		0.1965D 00	7	7	6	2	0	0	0	6
		0.1107D 00	7	7	6	2	2	0	4	6
		0.1911D 00	7	7	2	2	2	0	4	6
		-.3255D 00	7	7	4	1	1	0	2	6
		-.1147D 00	7	7	4	2	2	0	2	6
TOTAL=		0.1000000000000D 01								

Table VI. (Continued)

#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
2	6									
		0.1266D 00	1	7	4	1	1	0	2	6
		-.1973D 00	3	7	5	1	1	0	2	6
		0.1281D 00	3	7	5	2	2	0	4	6
		0.1497D 00	5	7	6	0	0	0	0	6
		0.3055D 00	7	7	6	3	3	0	6	6
		0.2193D 00	3	7	2	2	2	0	4	6
		-.1521D 00	3	7	2	3	3	0	6	6
		0.4292D 00	7	7	6	0	0	0	0	6
		-.1478D 00	5	7	4	1	1	0	2	6
		-.2913D 00	7	7	6	1	1	0	2	6
		0.1804D 00	7	7	6	2	2	0	2	6
		0.1929D 00	7	9	6	0	0	0	0	6
		-.3343D 00	7	7	2	2	2	0	4	6
		-.1131D 00	7	7	2	4	4	0	8	6
		0.2447D 00	7	7	4	1	1	0	2	6
		0.1001D 00	7	7	4	3	1	0	2	6
		-.1096D 00	7	9	4	1	1	0	2	6
TOTAL=		0.1000000000000000D 01								

Table VII. Electric quadrupole transitions for the N = 84 nuclei with a residual pairing interaction. The values of $B(E2;i \to f)$ and $\langle f||M(E2)||i\rangle$ are given for $e_n^{eff} = .2$ e. The upper number of each pair is the B(E2) value

i	f	$^{140}_{56}Ba_{84}$	$^{142}_{58}Ce_{84}$	$^{144}_{60}Nd_{84}$	$^{146}_{62}Sm_{84}$	$^{148}_{64}Gd_{84}$
2_1	0_1	.0782 -.6253	.0904 .6723	.1016 .7128	.1171 .7651	.1283 .8009
2_2	0_1	.0015 .0852	.0012 -.0787	.0008 .0647	.0003 -.0360	.0001 .0205
	2_1	.0714 -.2673	.0746 -.2731	.0802 .2832	.0839 -.2896	.0908 .3013
	4_1	.0002 -.0103	.0003 .0133	.0004 .0143	.0005 -.0165	.0005 -.0167
0_2	2_1	.0641 .1132	.0715 .1196	.0833 -.1290	.0920 -.1356	.1019 -.1427
	2_2	.0081 .0402	.0140 .0529	.0193 .0621	.0360 -.0848	.0448 .0947
3_1	2_1	.0000 -.0006	.0000 .0029	.0001 .0086	.0002 .0174	.0004 -.0229
	4_1	.0255 -.1408	.0302 .1532	.0354 -.1659	.0431 -.1831	.0489 -.1950
	2_2	.0523 -.2706	.0634 -.2978	.0726 .3187	.0852 -.3453	.0932 -.3611
2_3	0_1	.0099 .2228	.0142 .2668	.0185 .3039	.0262 -.3617	.0307 .3915
	2_1	.0097 -.0985	.0137 .1173	.0182 .1350	.0240 -.1548	.0275 .1658
	4_1	.0088 .0699	.0111 .0785	.0141 -.0884	.0190 .1027	.0226 .1120
	2_2	.0007 -.0256	.0004 .0190	.0002 -.0153	.0000 -.0036	.0000 -.0009
	0_2	.0009 .0656	.0014 -.0846	.0020 .0995	.0038 -.1374	.0047 .1531
	3_1	.0239 -.1306	.0229 .1280	.0216 .1241	.0186 -.1153	.0183 -.1142

$B(E2;i \to f)$

Table VIII. Magnetic dipole transitions for N = 84 nuclei with a residual pairing interaction. The values of $B(M1; i \to f)$ and $\langle f||M(M1)||i\rangle$ are given for $g_S = .7g_S^{free} = -2.68$, $g_\ell = 0$, $g_R = Z/A$. The upper number of each pair is the B(M1) value

i	f	^{140}Ba	^{142}Ce	^{144}Nd	^{146}Sm	^{148}Gd
2_1	0_2	0.0 0.0	0.0 0.0	0.0 0.0	0.0 0.0	0.0 0.0
2_2	0_1	0.0 0.0	0.0 0.0	0.0 0.0	0.0 0.0	0.0 0.0
	2_1	.0104 .1020	.0185 .1361	.0267 -.1635	.0434 .2083	.0503 -.2243
	4_1	0.0 0.0	0.0 0.0	0.0 0.0	0.0 0.0	0.0 0.0
0_2	2_1	0.0 0.0	0.0 0.0	0.0 0.0	0.0 0.0	0.0 0.0
	2_2	0.0 0.0	0.0 0.0	0.0 0.0	0.0 0.0	0.0 0.0
3_1	2_1	.0495 .2632	.0587 .2869	.0636 .2985	.0734 .3201	.0755 -.3250
	4_1	.0528 .2026	.0527 -.2025	.0520 .2011	.0475 .1922	.0459 .1888
	2_2	.1225 .4141	.1350 .4348	.1439 -.4488	.1550 .4658	.1591 .4719
2_3	0_1	0.0 0.0	0.0 0.0	0.0 0.0	0.0 0.0	0.0 0.0
	2_1	.2810 -.5301	.3024 .5499	.3142 .5605	.3344 -.5783	.3430 .5857
	4_1	0.0 0.0	0.0 0.0	0.0 0.0	0.0 0.0	0.0 0.0
	2_2	.2144 .4630	.2379 -.4878	.2394 .4893	.2314 .4811	.2214 .4705
	0_2	0.0 0.0	0.0 0.0	0.0 0.0	0.0 0.0	0.0 0.0
	3_1	.0174 -.1115	.0308 .1484	.0442 .1777	.0728 -.2280	.0845 -.2457

$B(M1; i \to f)$

e_n^{eff} = .5 e used by Vanden Berghe, but is consistent with our contention that we are treating the basis truncation more correctly and so should need less effective charge (see Appendix B). We used $g_S = .7\ g_S^{free}$ for the value of the spin gyromagnetic ratio for the extra-core particles. This value has been found to work well by other authors (27), and is supposed to take into account the fact that we have not included magnetic interactions or polarization effects (27). The calculated electric quadrupole and magnetic dipole moments for the first 2^+ state and the value of $B(E2;2_1^+ \rightarrow 0_1^+)$ are compared with the presently known experimental values in Table IX. The agreement is remarkably good.

The calculated branchings and mixing ratios are compared with the experimental findings of Alquist (47) et al. in Table Xa. The results of a previous calculation (47) using the Vanden Berghe wavefunctions (in a slightly modified fashion) are also included. The agreement is good overall, and a few discrepancies found in the previous calculation seem to be remedied. In particular the mixing ratio of the $3_1^+ \rightarrow 4_1^+$ transition is here predicted to have the correct sign and magnitude, and the theoretical prediction of the mixing ratio for the $2_3^+ \rightarrow 2_1^+$ transition is greatly improved. The only really glaring discrepancy is the fact that only the $3_1^+ \rightarrow 4_1^+$ decay was seen experimentally whereas the model predicts that this state should decay predominantly to the 2_1^+ state. We also predict some strength from the 2_2 to the ground state, but Alquist apparently saw none.

In Table Xb we compare the experimental and theoretical mixing ratios for ^{142}Ce, using the data of Bassinger et al. (75). The agreement

Table IX. Theoretical and known experimental values of $B(E2;2_1^+ \rightarrow 0_1^+)$, $Q(2_1^+)$ and $\mu(2_1^+)$ for the N = 84 nuclei. The theoretical values are calculated for the case of a residual pairing interaction and the parameters are as indicated for Tables VIII and IX

	$B(E2;2_1 \rightarrow 0_1)$		$Q(2_1)$		$\mu(2_1)$	
	Theory	Experiment	Theory	Experiment	Theory	Experiment
^{140}Ba	.0782		-.36		.3012	
^{142}Ce	.0904	.091 ± .002	-.40	-.12 ± .09	.260	
^{144}Nd	.1017	.102 ± .003	-.42	-.39 ± .21	.233	.26 ± .04
^{146}Sm	.1171		-.46		.171	
^{148}Gd	.1283		-.48		.154	

Table Xa. Experimental and theoretical branchings and mixing ratios for ^{140}Ba using the residual pairing interaction. The parameters are given in Tables VIII and IX

Sequence	δ_{calc}	δ_{exp}	Ref. (47)	Calc.	Exp.	Ref. (47)
$2_2 \rightarrow 0_1$	-	-	-	17(0,17)	0	12(0,12)
2_1	-1.98	-1.1	-1.7	83(17,66)	100(45,55)	88(23,65)
4_2	-	-	-	0(0,0)	0	0
$0_2 \rightarrow 2_1$	-	-	-	100(0,100)	100(0,100)	100(0,100)
2_2	-	-	-	0	0	0
$3_1 \rightarrow 2_1$	-2.83	-	.14	73(73,0)	0	57(55,2)
4_1	- .42	- .51	+ .37	21(18,3)	100(80,20)	33(29,4)
2_2	-	-	.15	6(6,0)	0	10(9.6,.04)
$2_3 \rightarrow 0_1$	-	-	-	19(0,19)	19(0,19)	18(0,18)
2_1	.216	.16	.61	78(75,3)	81(79,2)	81(59,22)
4_1	-	-	-	1(0,1)	0	1(0,1)
2_2	.186	-	- .13	2(2,0)	0	0
3_1	.05	-	- .01	0	0	0
0_2	-	-	-	0	0	0

Table Xb. Experimental and theoretical branchings and mixing ratios for ^{142}Ce using the residual pairing interaction. The parameters are given in Tables VIII and IX

Transition	δ_{exp}	δ_{theory}	% E2 Exp.	% E2 Theory
$2_2 \rightarrow 2_1$	$-.61 \pm .18$	-1.5	27	69
$2_3 \rightarrow 2_1$	$.09 \pm .06$	.24	~ 0	5.5 %
$3_1 \rightarrow 4_1$	$-1.01 \begin{matrix} -2.08 \\ +.46 \end{matrix}$	-.54	51	23
$1_1 \rightarrow 2_1$	$.01 \pm .04$	6.36	0	97
$1_2 \rightarrow 2_1$	$-.35 \pm .05$	-.45	11 ± 3	17
$1_3 \rightarrow 2_1$	$-1.06 \pm .13$	-1.203	53 ± 6	59
$1_4 \rightarrow 2_1$	$.05 \pm .15$	-.25	<4	6
$2_4 \rightarrow 2_1$	$-.55 \pm .27$	-.61	24 ± 16	27

is seen to be very good in both the magnitudes and signs, with one or two exceptions. In particular we predict that the $1_1 \rightarrow 2_1^+$ transition should be almost entirely E2 and it was observed to be entirely M1. Since the parity of this state was undetermined experimentally, this would seem to indicate that the experimental state we have identified as the 1_1^+ is probably not a 1^+ level. Rather is would appear that the 1^+ state was not seen and that this experimental state is a 1^- and is not therefore described by the model. From the angular distributions above both would indicate L = 1.

Nuclei With N = 84. Realistic Residual Interaction

Basis selection

The same procedure as described earlier for the pairing residual interaction was followed. We find that the realistic matrix elements of Goodman et al. (70) do not produce as much splitting of the single particle levels as the pairing force does. The effect of this is to force one to take lower values for E_{sp} (the diagonalized single particle energy cutoff energy) to obtain a manageable-size basis. Using E_{sp} = 4.0 MeV we varied E_{cut} and found the energies converged for E_{cut} = 5.8 MeV. These results are shown in Figure 12 where the second and eighth spin zero states are shown.

Fits to experimental spectra

As in the case of a residual pairing interaction, we attempted to determine $\hbar\omega$ and ξ by fitting to the experimental spectra here using the realistic residual interaction. For this interaction the fitting

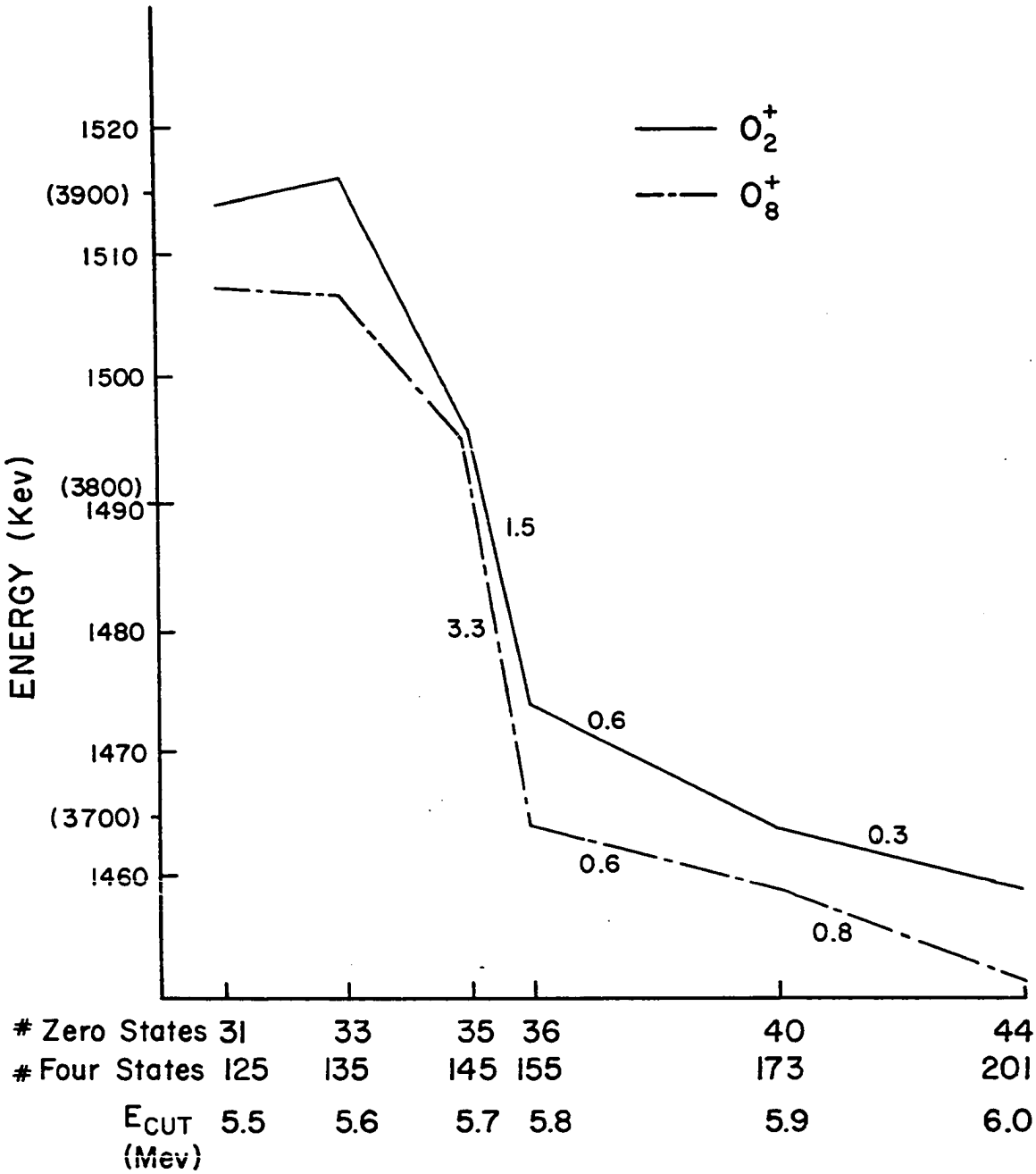

Figure 12. Convergence of the energy levels for the N = 84 nuclei using a realistic residual interaction. The energies of the second and eighth spin zero state are shown. The percent change in energy as the basis increases is indicated.

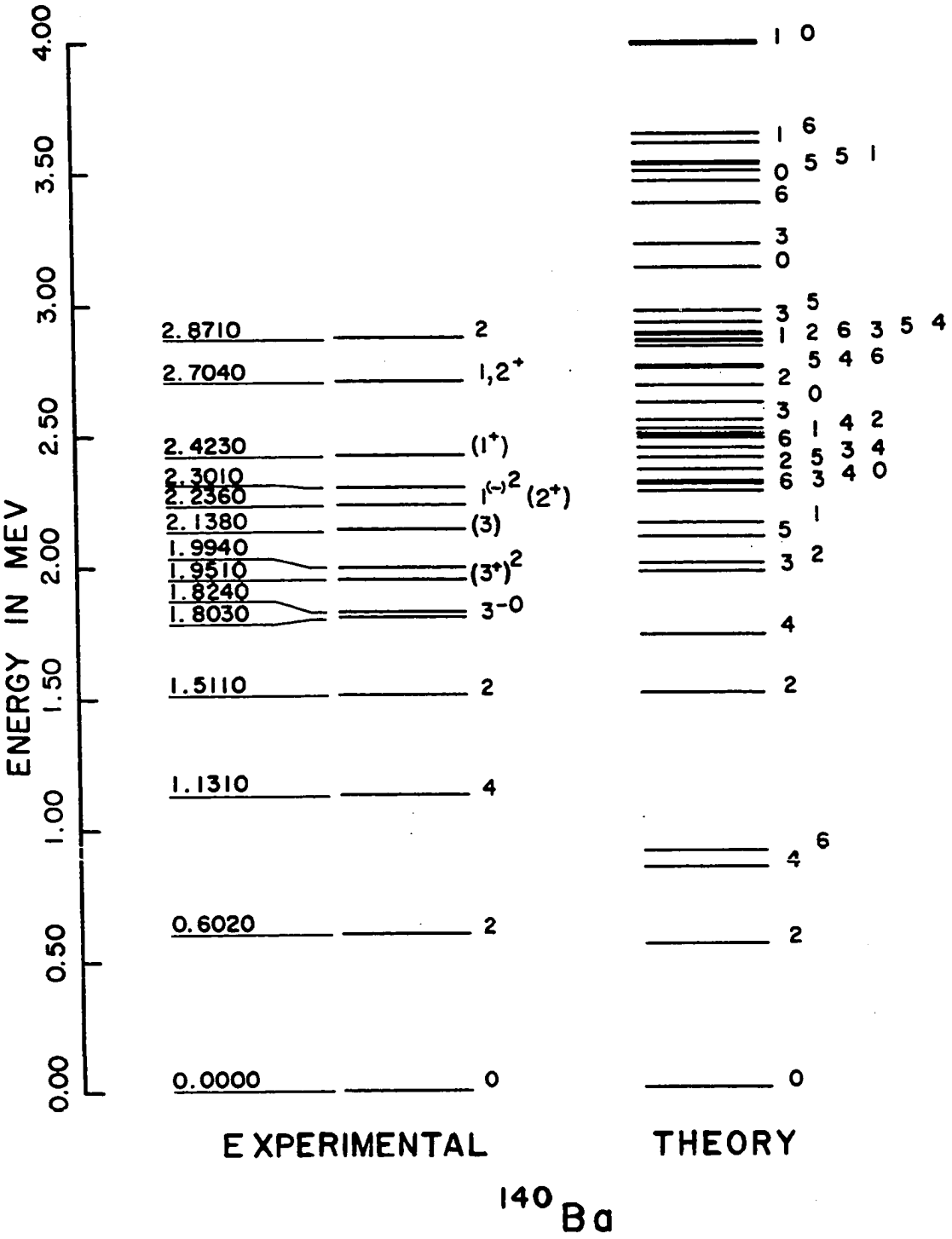

Figure 13. Theoretical and experimental energy levels of ^{140}Ba. The theoretical levels were calculated using a realistic residual interaction. The model parameters were $\hbar\omega = 1.920$ and $\xi = 0.445$. The theoretical (positive parity) levels are only shown for $I \leq 6$.

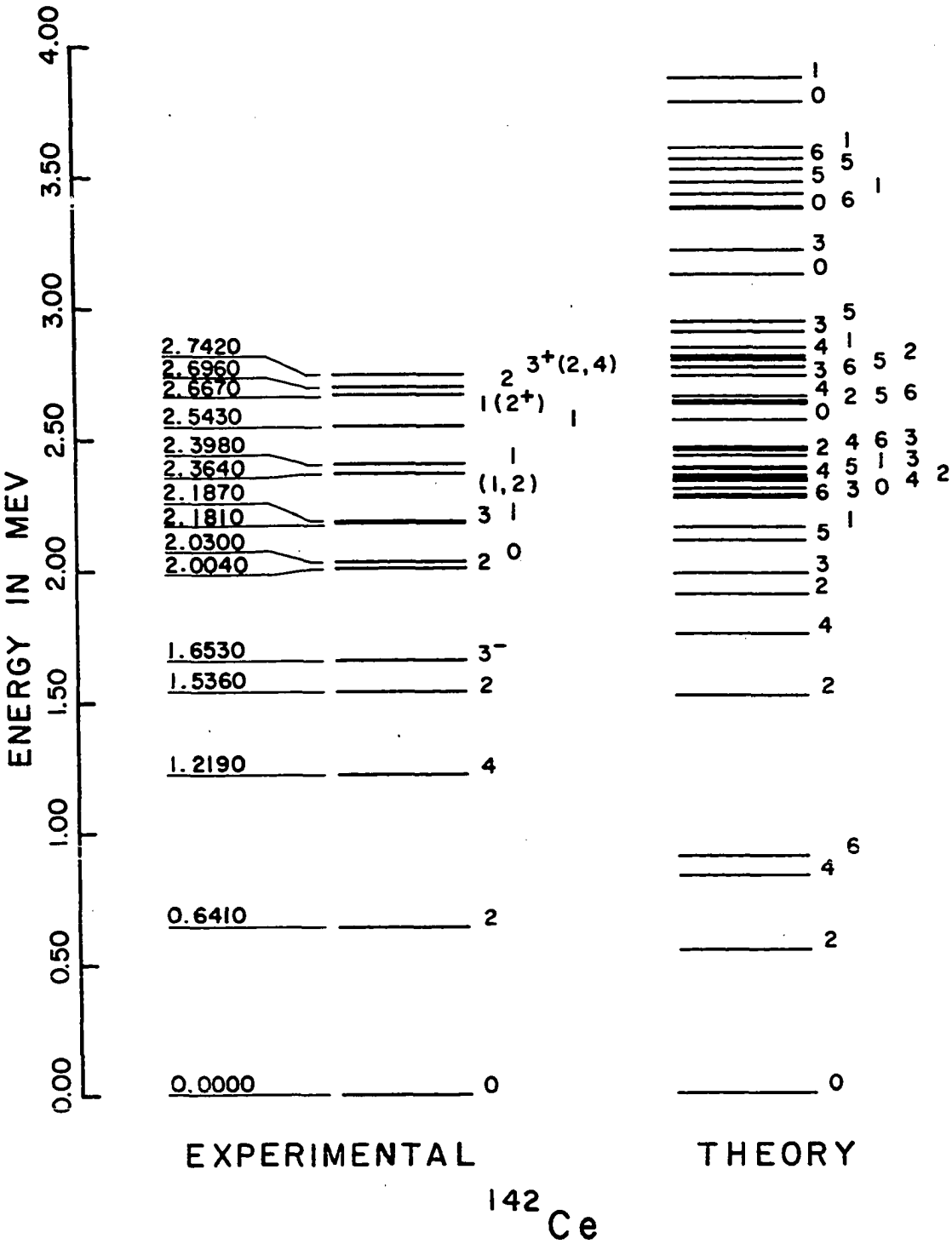

Figure 14. Theoretical and experimental energy levels of ^{142}Ce. The theoretical levels were calculated using a realistic residual interaction. The model parameters were $\hbar\omega = 1.879$ and $\xi = 0.787$. The theoretical (positive parity) levels are only shown for $I \leq 6$.

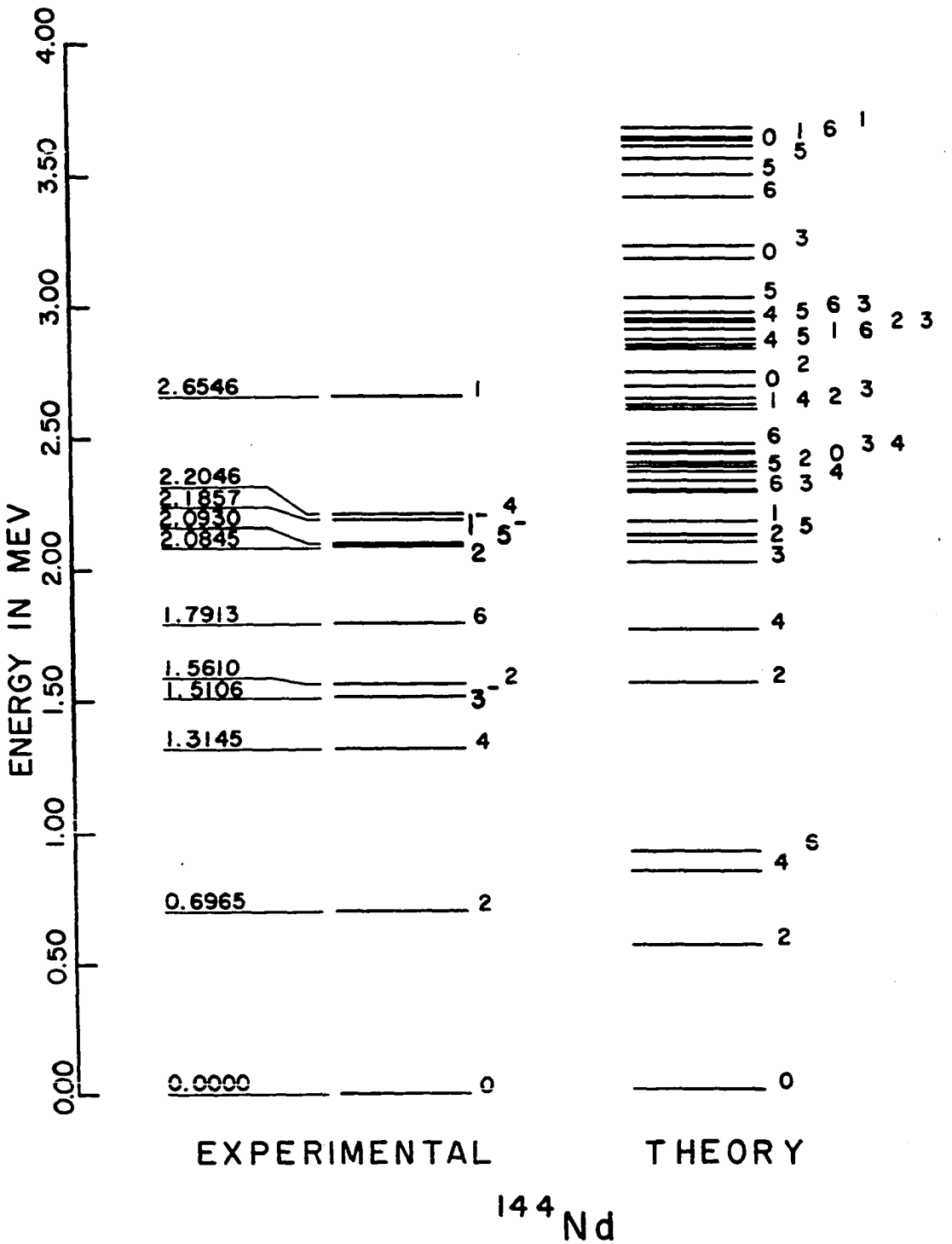

Figure 15. Theoretical and experimental energy levels of ^{144}Nd. The theoretical levels were calculated using a realistic residual interaction. The model parameters were $\hbar\omega = 2.037$ and $\xi = 0.378$. The theoretical (positive parity) levels are only shown for $I \leq 6$.

turned out to be considerably more difficult. The results for ^{140}Ba, ^{142}Ce, and ^{144}Nd are shown in Figures 13 to 15 and one notes that the theoretical energy-spectra clearly are not as satisfactory as those we found with the pairing interaction. Although we were able to reproduce the spacing of the spin two levels respectively well, and do reasonably well for the spin three and spin one levels, the spin four and spin six states are seen to be very poorly represented. They are consistently predicted to be much lower in energy than they are seen experimentally. This would appear to be due to the lack of pairing strength in this interaction which we noted when we discussed the basis selection. The single particle levels are simply not split nearly enough by this interaction.

The eigenvectors for some of the ^{140}Ba states are shown in Table XI. On comparing these with the corresponding eigenvectors for the pairing interaction case (Table VI) one can see that for the realistic residual interaction the states are considerably less mixed (more pure) than for the pairing interaction.

Electromagnetic properties

Some calculated B(E2) values for ^{140}Ba are given in Table XII, and the corresponding values found when using the pairing residual interaction are also shown for comparison. We also indicate the static quadrupole and dipole moment in both calculations.

The pairing results were in remarkably good agreement with experiment; the realistic interaction results are very disappointing. Transitions which are weak are predicted to be strong, and vice versa.

Table XI. Eigenvectors for ^{140}Ba calculated using the realistic interaction

#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
1	0									
		0.1170D 00	1	1	0	0	0	0	0	0
		0.1837D 00	3	3	0	0	0	0	0	0
		0.2596D 00	5	5	0	0	0	0	0	0
		0.8943D 00	7	7	0	0	0	0	0	0
		0.1935D 01	9	9	0	0	0	0	0	0
		0.1234D 00	3	7	2	1	1	0	2	0
		-.1339D 00	13	13	0	0	0	0	0	0
TOTAL=		0.9999999999999982D 00								

#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
2	0									
		0.2470D 00	1	1	0	0	0	0	0	0
		0.5468D 00	3	3	0	0	0	0	0	0
		-.1529D 00	7	7	0	0	0	0	0	0
		0.1723D 00	9	9	0	0	0	0	0	0
		-.1823D 00	3	7	2	1	1	0	2	0
		-.1135D 00	13	13	0	0	0	0	0	0
		-.6989D 00	7	7	2	1	1	0	2	0
		0.1330D 00	7	7	2	2	2	0	2	0
TOTAL=		0.9999999999999976D 00								

#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
1	1									
		0.2336D 00	5	7	1	0	0	0	0	1
		-.9343D 00	7	9	1	0	0	0	0	1
		0.1329D 00	7	9	1	1	1	0	2	1
		0.1116D 00	7	9	3	1	1	0	2	1
		-.1414D 00	7	7	2	1	1	0	2	1
TOTAL=		0.9999995999999979D 00								

Table XI. (Continued)

#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
2	1									
		0.1031D 00	1	5	2	1	1	0	2	1
		-.1640D 00	5	7	1	0	0	0	0	1
		-.1857D 00	7	9	1	0	0	0	0	1
		0.2963D 00	3	7	2	1	1	0	2	1
		0.8755D 00	7	7	2	1	1	0	2	1
		-.1095D 00	7	7	4	2	2	0	4	1
TOTAL=		0.99999999999982D 00								

#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
1	2									
		-.1053D 00	1	5	2	0	0	0	0	2
		-.2955D 00	3	7	2	0	0	0	0	2
		-.9045D 00	7	7	2	0	0	0	0	2
		-.1352D 00	7	7	2	1	1	0	2	2
TOTAL=		0.99999999999991D 00								

#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
2	2									
		0.1474D 00	1	3	2	0	0	0	0	2
		-.1586D 00	1	5	2	0	0	0	0	2
		-.1077D 00	3	3	0	1	1	0	2	2
		-.1362D 00	3	3	2	0	0	0	0	2
		-.1159D 00	5	5	0	1	1	0	2	2
		-.3904D 00	7	7	0	1	1	0	2	2
		-.7042D 00	3	7	2	0	0	0	0	2
		0.1213D 00	3	7	2	1	1	0	2	2
		0.1331D 00	5	7	2	0	0	0	0	2
		-.1015D 00	5	9	2	0	0	0	0	2
		0.3584D 00	7	7	2	0	0	0	0	2
		0.1325D 00	7	7	4	1	1	0	2	2
TOTAL=		0.99999999999963D 00								

Table XI. (Continued)

#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
1	3									
		-.8464D 00	3	7	3	0	0	0	0	3
		0.1111D 00	7	9	3	0	0	0	0	3
		-.3659D 00	7	7	2	1	1	0	2	3
		-.2834D 00	7	7	4	1	1	0	2	3
TOTAL=		0.9999999999999968D 00								

#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
2	3									
		-.1829D 00	3	7	3	0	0	0	0	3
		0.1160D 00	5	7	3	0	0	0	0	3
		-.9358D 00	7	9	3	0	0	0	0	3
		0.1038D 00	7	9	5	1	1	0	2	3
		0.1628D 00	7	7	2	1	1	0	2	3
TOTAL=		0.1000000000000000D 01								

#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
1	4									
		-.1576D 00	1	7	4	0	0	0	0	4
		0.1178D 00	3	7	4	0	0	0	0	4
		-.1092D 00	5	7	4	0	0	0	0	4
		0.9552D 00	7	7	4	0	0	0	0	4
TOTAL=		0.9999999999999963D 00								

#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
2	4									
		0.4681D 00	1	7	4	0	0	0	0	4
		-.2345D 00	3	5	4	0	0	0	0	4
		-.7088D 00	3	7	4	0	0	0	0	4
		0.1755D 00	5	7	4	0	0	0	0	4
		-.2534D 00	7	7	6	1	1	0	2	4
		0.1984D 00	7	7	4	0	0	0	0	4
		-.1208D 00	7	9	4	0	0	0	0	4
TOTAL=		0.9999999999999941D 00								

Table XI. (Continued)

#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
1	5									
		0.9416D 00	3	7	5	0	0	0	0	5
		-.1426D 00	3	7	5	1	1	0	2	5
		0.2397D 00	7	7	6	1	1	0	2	5
TOTAL=		0.9999999999999959D 00								
#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
2	5									
		0.1112D 00	5	7	5	0	0	0	0	5
		-.9743D 00	7	9	5	0	0	0	0	5
TOTAL=		0.9999999999999999D 00								
#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
1	6									
		-.2310D 00	5	7	6	0	0	0	0	6
		0.9549D 00	7	7	6	0	0	0	0	6
		-.1204D 00	7	7	6	1	1	0	2	6
TOTAL=		0.9999999999999969D 00								
#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
2	6									
		-.1396D 00	3	9	6	0	0	0	0	6
		0.8204D 00	5	7	6	0	0	0	0	6
		0.2049D 00	7	7	6	0	0	0	0	6
		-.1225D 00	7	7	6	1	1	0	2	6
		-.4373D 00	7	9	6	0	0	0	0	6
		0.1435D 00	7	7	4	1	1	0	2	6
TOTAL=		0.9999999999999966D 00								

Table XII. Comparison between the theoretical B(E2) values of ^{140}Ba calculated using the realistic interaction and pairing interaction

i	f	Realistic	Pairing
2_1	2_1	.0078 .08841	.0448 -.2116
2_1	0_1	.0008 -.0615	.0782 -.6253
2_2	2_1	.0025 -.0495	.0714 -.2673
	0_1	.0192 -.3101	.0015 .0852
	4_1	.0020 .0335	.0002 -.0103
3_1	2_1	.0141 -.1402	.0000 -.0006
	4_1	.0157 .1104	.0255 -.1408
	2_2	.0011 .0393	.0523 -.2706
	$Q(2_1^+)$ =	.149	-.360
	$\mu(2_1^+)$ =	-.753	.301

Similarly, the quadrupole and dipole moments have both the wrong sign and magnitude.

Since the realistic matrix elements are considerably more expensive to use and produced such dismal results for ^{140}Ba, we did not continue to calculate transitions.

Nuclei With N = 80. Pairing Interaction

Basis selection

The single particle basis space was taken to be the hole states: $(3s_{1/2})^{-1}$, $(2d_{1/2})^{-1}$, $(2d_{5/2})^{-1}$, $(1g_{7/2})^{-1}$, and $(1h_{11/2})^{-1}$. The pairing strength was taken as G = .184 MeV and the single particle energies $E_i - E_{3/2}$ were $E_{1/2} = .360$, $E_{5/2} = 1.59$, $E_{7/2} = 2.39$, and $E_{11/2} = .410$ (MeV). These single particle energies were taken from a calculation for the N = 81 nuclei by Heyde and Brussaard (34). The energies reported there did not vary much from nucleus to nucleus, so an average was used to obtain the indicated values. As with our N = 84 calculation, we first diagonalized the single particle plus residual particle-particle part of the Hamiltonian to obtain the two-particle coupled basis states, and then generated and diagonalized the spin zero matrices for up to four phonon states with $\hbar\omega = 1.2$ MeV and $E_{sp} \leq \hbar\omega$. The results, for increasing values of E_{cut} are given in Table XIII. The energies had converged by $E_{cut} = 6.8$ MeV, but the basis size was larger than we could handle in a reasonable time with our fitting routine, so we took $E_{cut} = 6.7$ MeV. This value does not strictly meet our previously stated criteria of stopping when the change in energies was 1% or less, but

Table XIII. Basis convergence for N = 80 using a pairing interaction

E_{cut}	#0	#4	0_2	0_3	0_4	0_6	0_8	0_{10}
5.9	28	104	1.566	1.973	2.501	3.573	4.215	4.424
6.0	33	130	1.578	1.968	2.519	3.550	4.084	4.320
6.1	36	146	1.575	1.967	2.356	3.507	3.998	4.261
6.7	41	163	1.545	1.961	2.366	3.428	3.928	4.219
6.8	50	206	1.517	1.960	2.351	3.377	3.815	4.153
7.0	54	233	1.517	1.960	2.350	3.377	3.815	4.153

they had converged to a variation of less than 2% so we expected them to be reasonably stable.

Fits to experimental spectra

The behavior of some of the lower positive parity energy levels as ξ is increased for a fixed value of $\hbar\omega$ is shown in Figure 16, and the systematics of the N = 80 experiment spectra are shown in Figure 17. It is interesting to note that the experimentally seen reversal of the ordering of the 2_2 and 0_2 doublet is reproduced in the theoretical levels as ξ is increased.

We fit the experimental spectra of ^{134}Xe (79, 80), ^{136}Ba (81, 82a, 82b), ^{138}Ce (83, 84), and ^{140}Nd (85, 86), and the results are shown in Figures 18 to 21. The fitted values of $\hbar\omega$ and ξ are displayed in Figure 22. They are not as smoothly varying as those seen for the N = 84 case, Figure 4. This occurs because the 4_1^+ level does not change smoothly as A increases, and the fitting program follows this behavior, being discontinuous at ^{134}Xe. The experimental spectra are seen to be reproduced well for the lower-lying states. These nuclei were treated in an earlier calculation by Meyer-Levy and Lopac (87). Our agreement with the experimental spectra is seen to be much better than theirs, as shown in Figure 23, where we compare both their spectra and ours with the experimental energy spectra for ^{140}Nd. (In fairness, the authors gave the energy levels only in a small figure without scale, so it was somewhat difficult to extract their exact values).

The eigenvectors for ^{136}Ba are given in Table XIV. It is interesting to note that the four phonon core states do not contribute

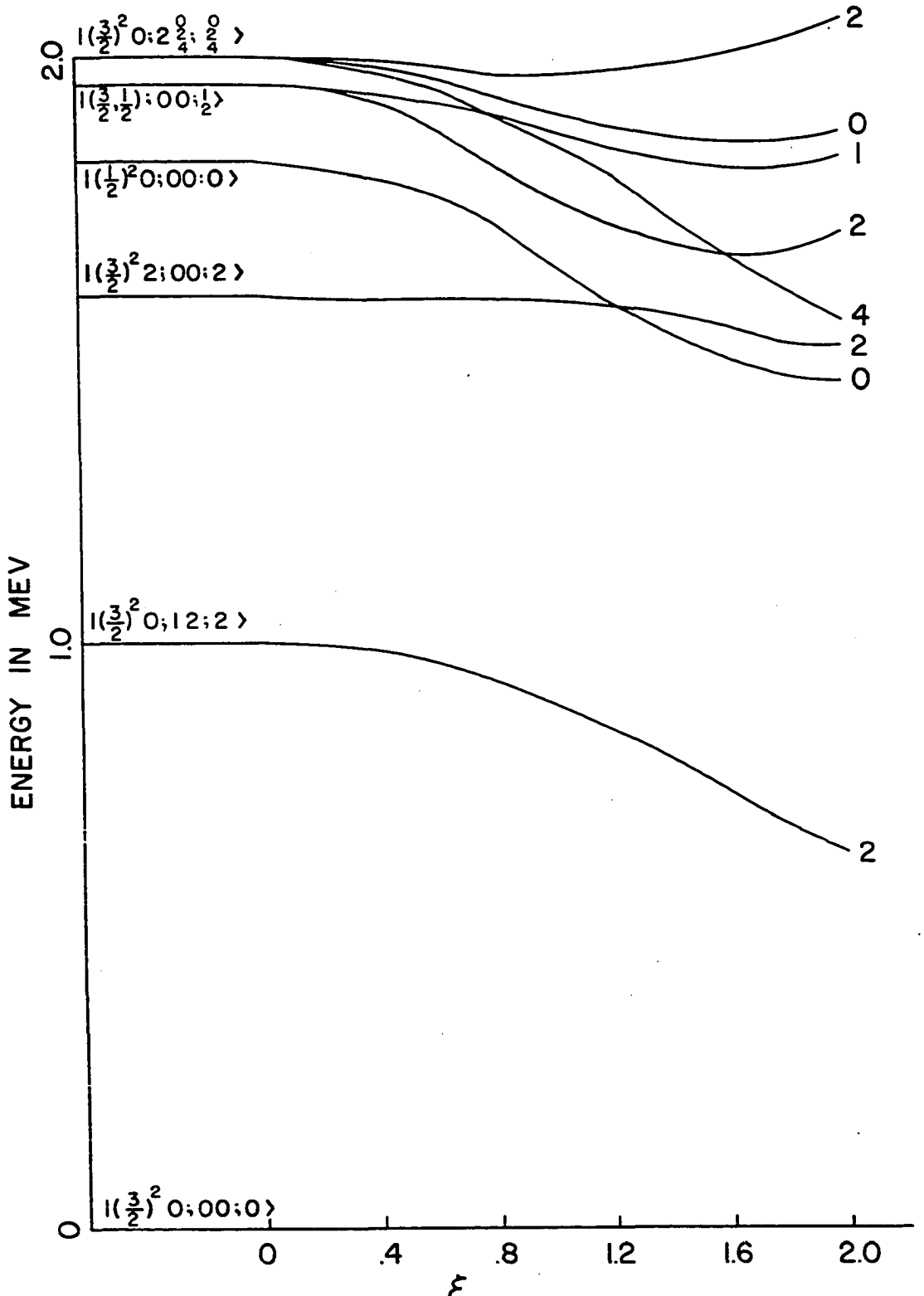

Figure 16. Behavior of the lower lying, theoretical positive parity states for the N = 80 nuclei shown as a function of increasing ξ for $\hbar\omega$ = 1.0 MeV.

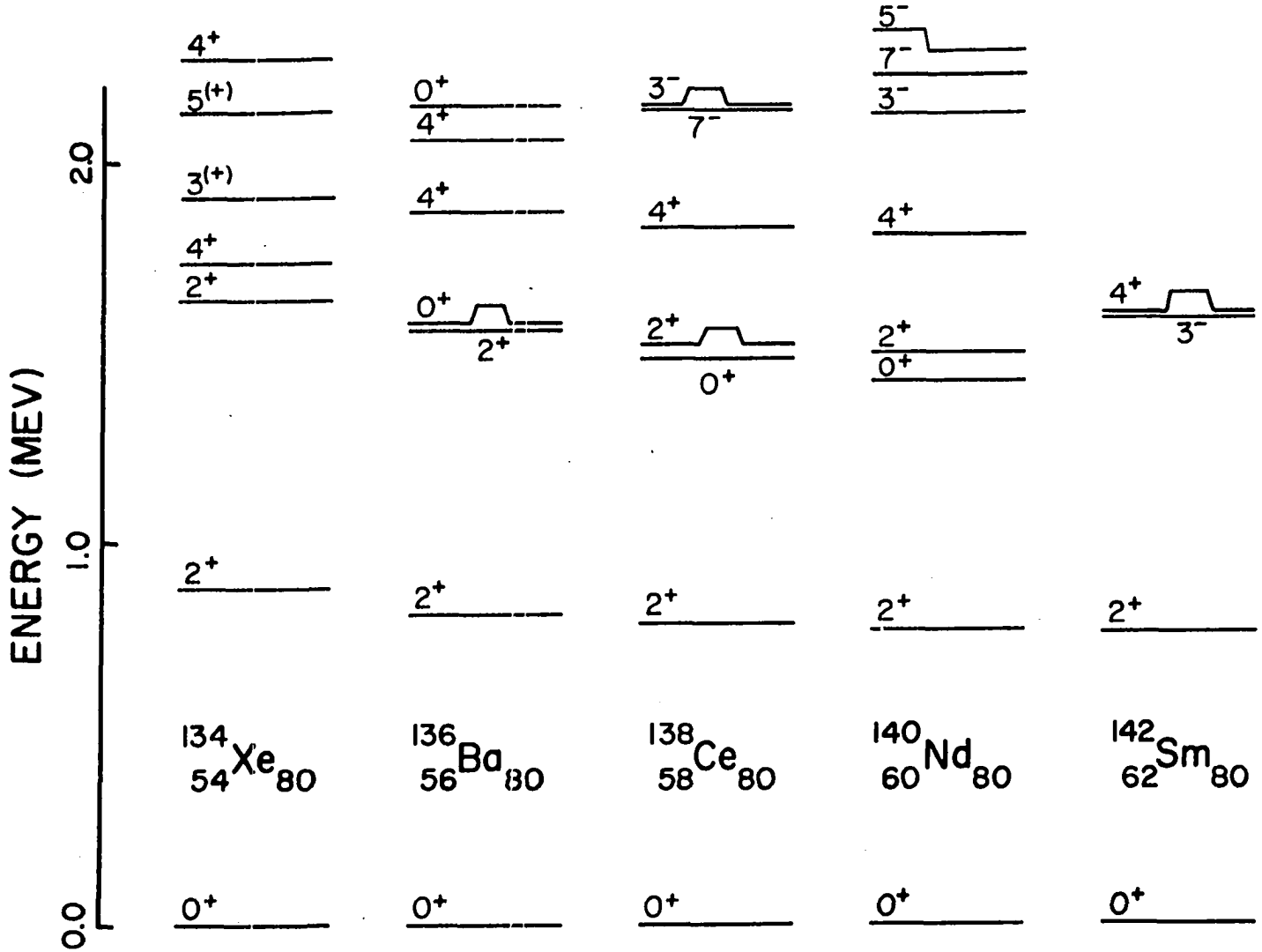

Figure 17. Systematics of low lying experimental energy levels for the N = 80 nuclei.

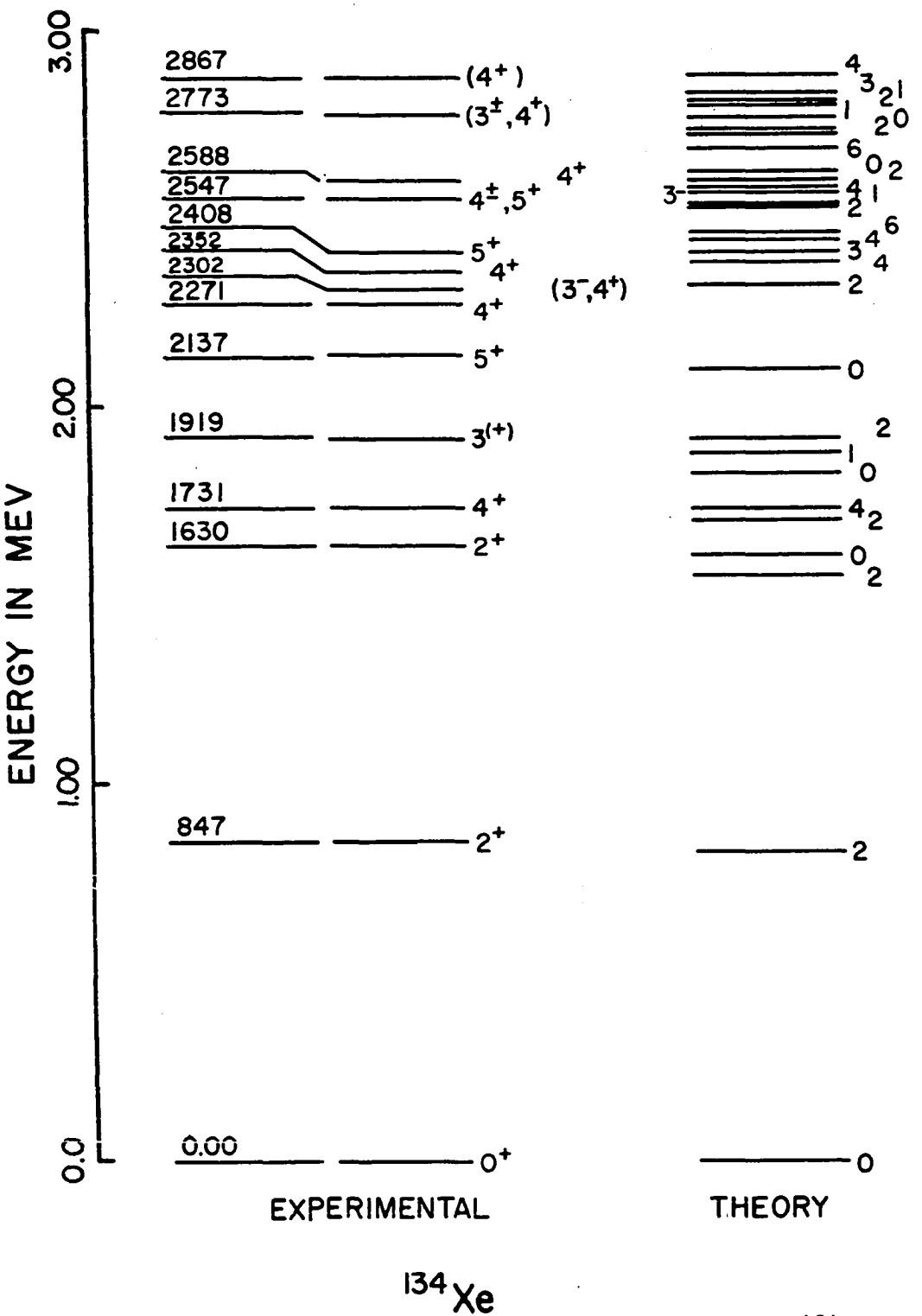

Figure 18. Theoretical and experimental energy levels of ^{134}Xe. The theoretical levels were calculated using a pairing residual interaction. Only those theoretical energy levels with $I \leq 6$ are shown. All have positive parity and the model parameters were $\hbar\omega$ = .942 MeV, ξ = 1.14.

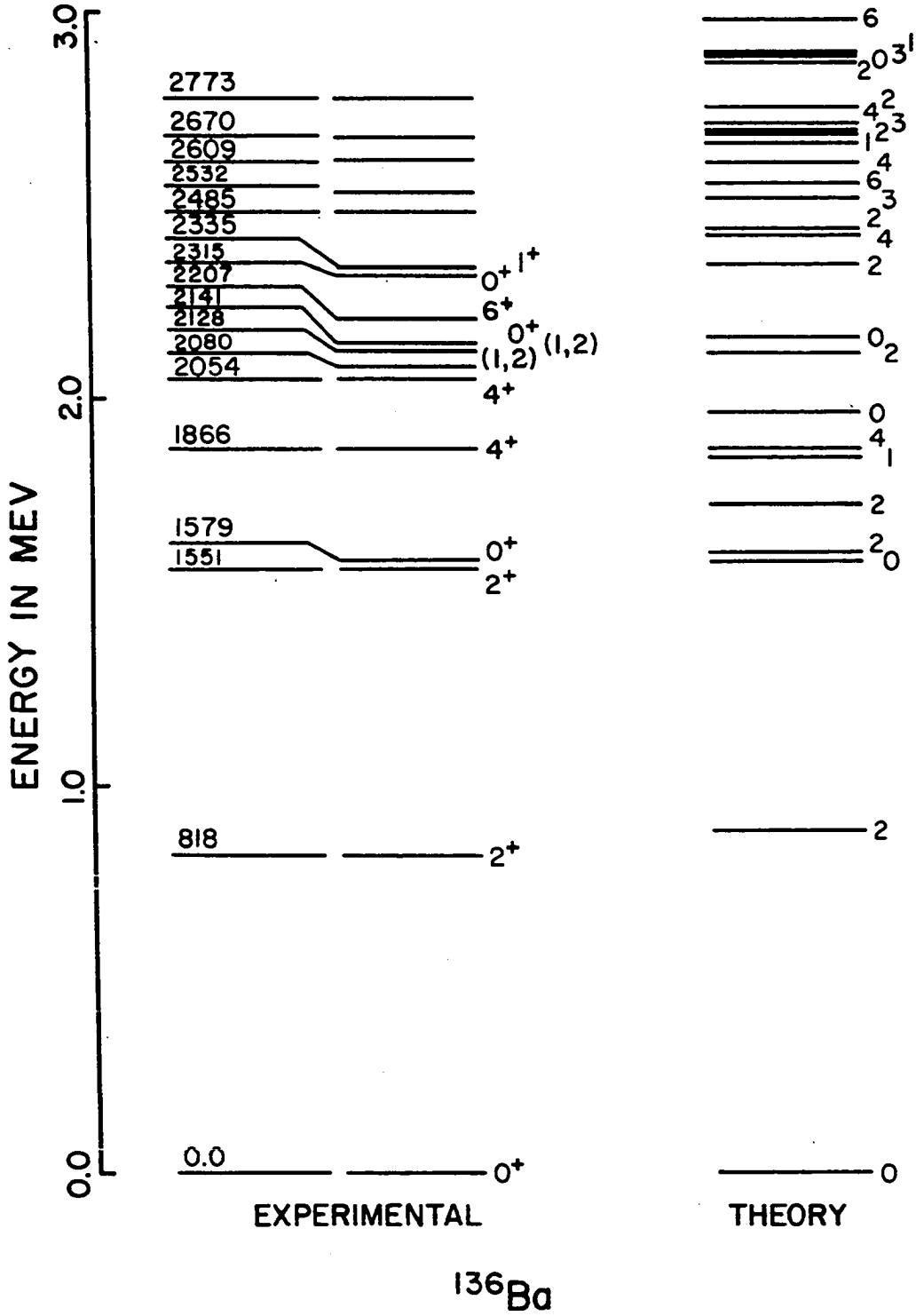

Figure 19. Theoretical and experimental energy levels of ^{136}Ba. The theoretical levels were calculated using a pairing residual interaction. Only those theoretical energy levels with $I \leq 6$ are shown. All have positive parity and the model parameters were $\hbar\omega = 1.1$ MeV, $\xi = 1.31$.

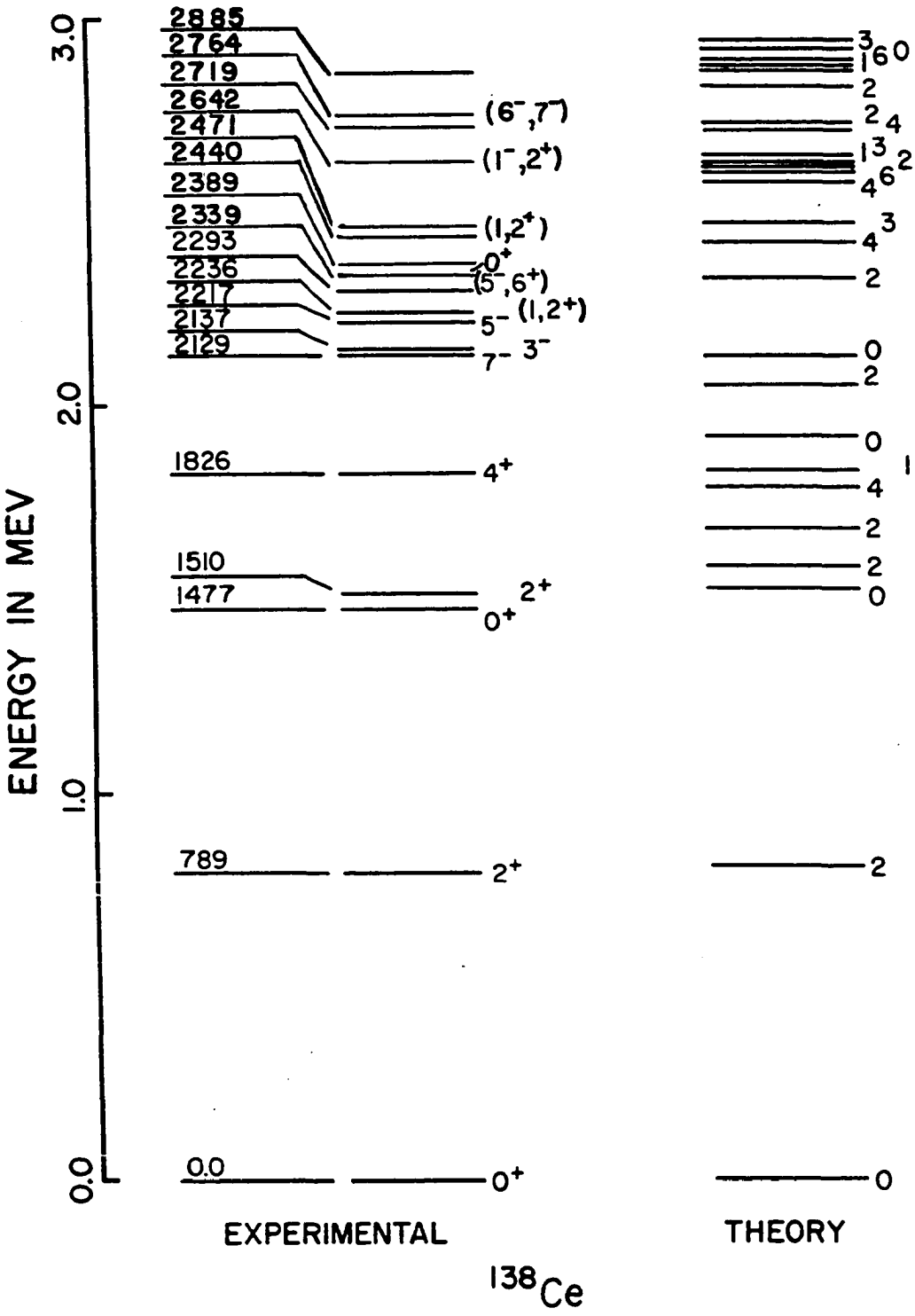

Figure 20. Theoretical and experimental energy levels of ^{138}Ce. The theoretical levels were calculated using a pairing residual interaction. Only those theoretical energy levels with $I \leq 6$ are shown. All have positive parity and the model parameters were $\hbar\omega = 1.077$ MeV, $\xi = 1.49$.

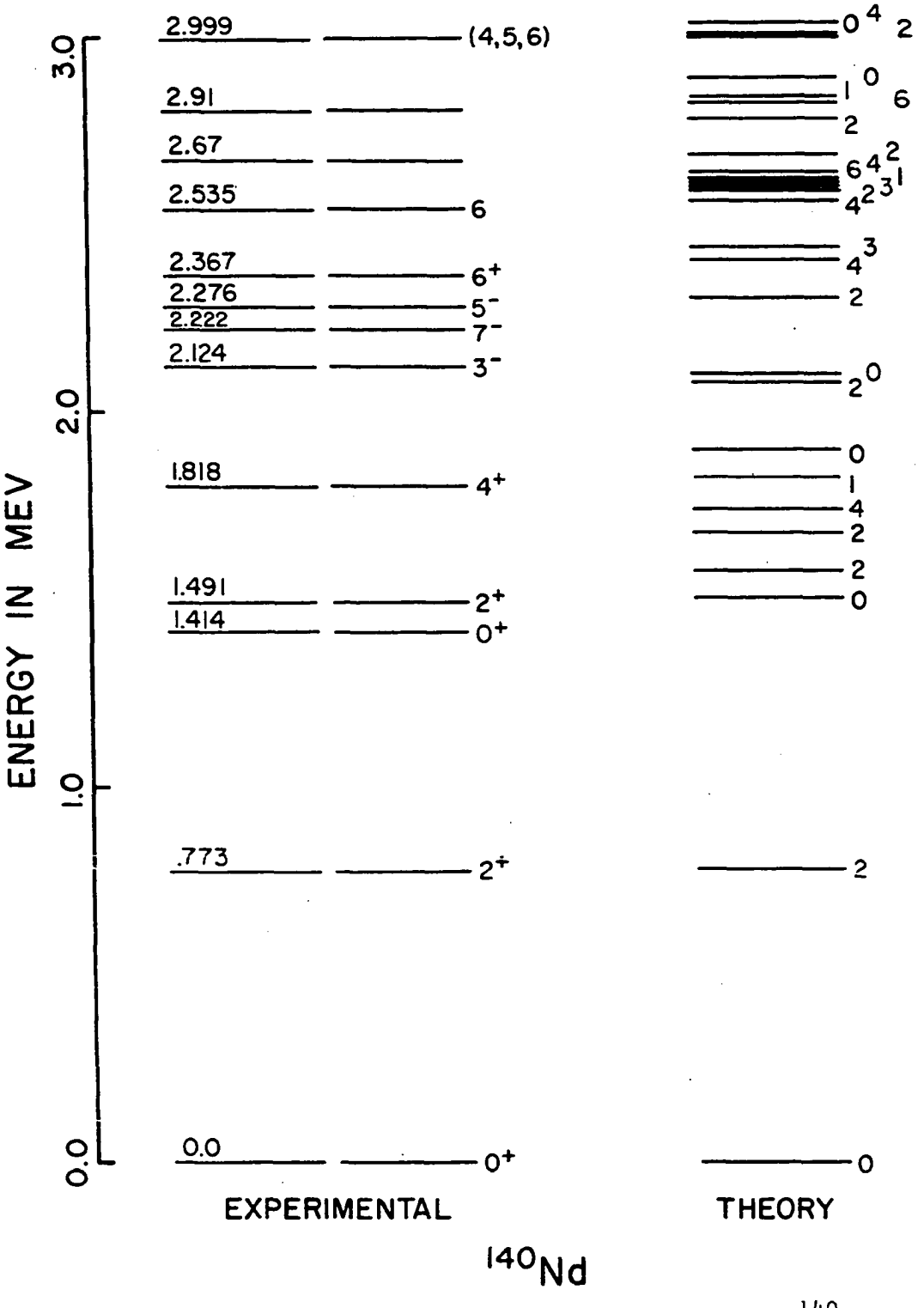

Figure 21. Theoretical and experimental energy levels of ^{140}Nd. The theoretical levels were calculated using a pairing residual interaction. Only those theoretical energy levels with $I \leq 6$ are shown. All have positive parity and the model parameters were $\hbar\omega = 1.056$ MeV, $\xi = 1.56$.

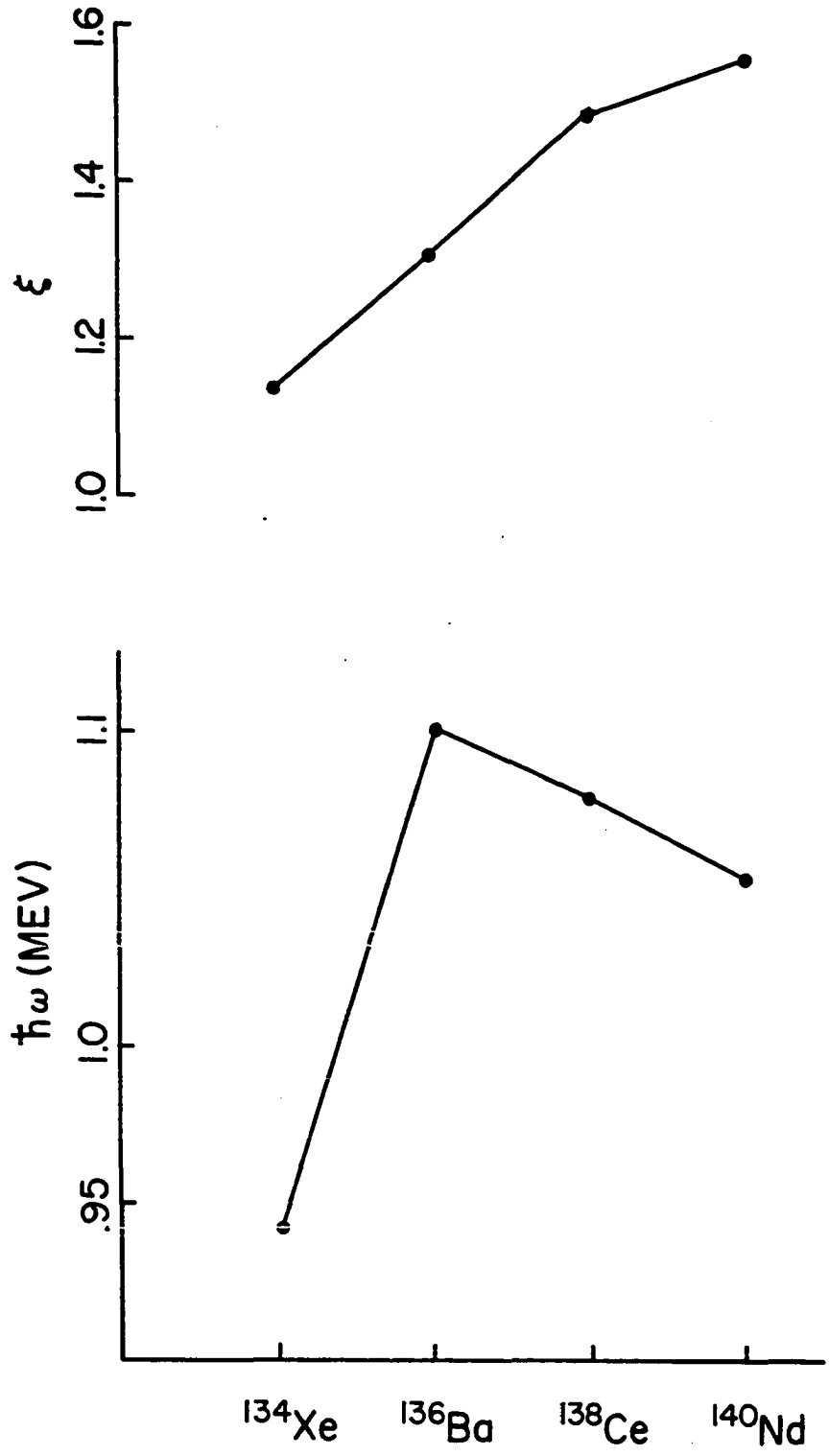

Figure 22. Values of the model parameters $\hbar\omega$ and ξ for the N = 80 nuclei are shown as a function of A.

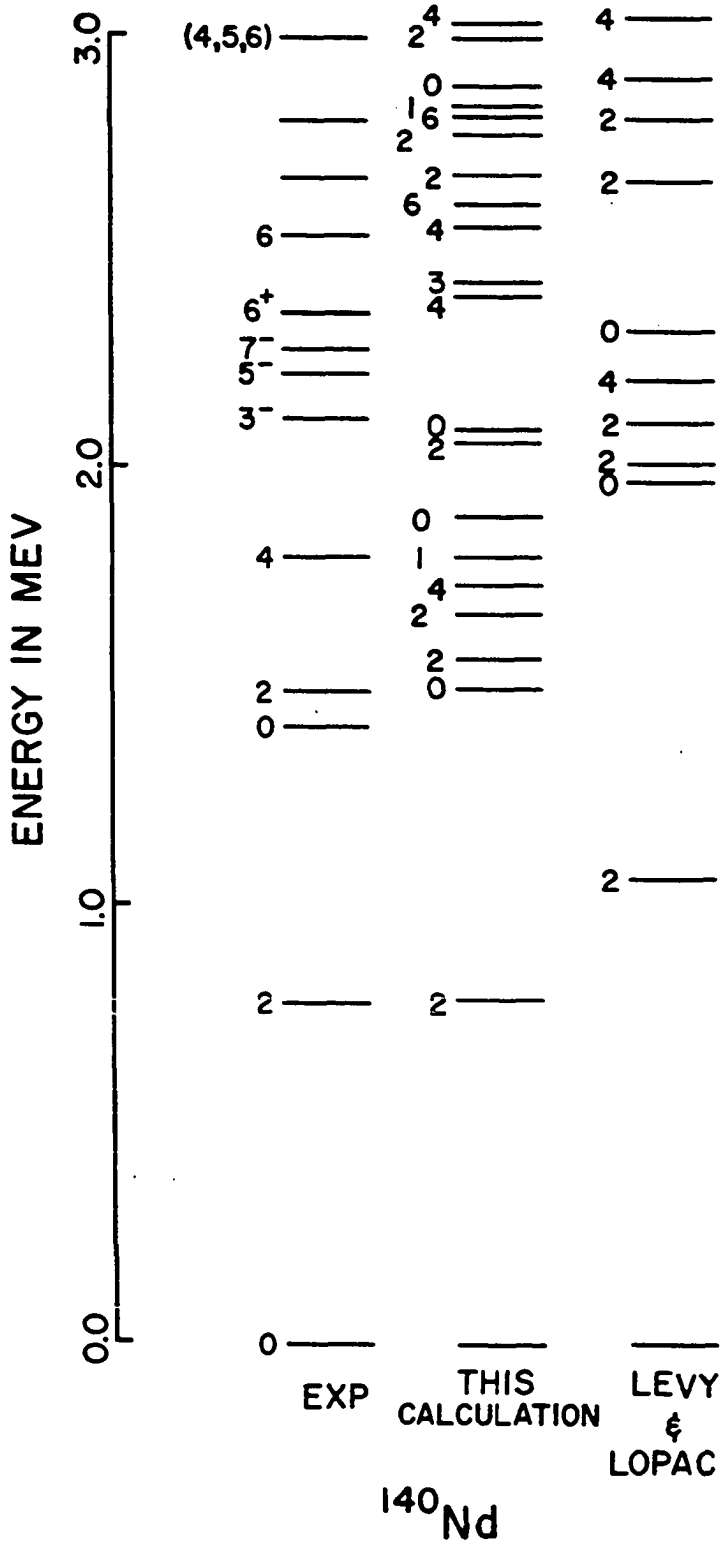

Figure 23. Comparison between the theoretical energy spectra predicted in this work and the theoretical spectra of Meyer-Levy and Lopac (87) for ^{140}Nd. For reference, the experimental levels are also shown.

Table XIV. Eigenvectors for ^{136}Ba using a pairing interaction

#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
1	0									
		0.3016D 00	1	1	0	0	0	0	0	0
		-.1666D 00	1	3	2	1	1	0	2	0
		0.6065D 00	3	3	0	0	0	0	0	0
		0.2258D 00	5	5	0	0	0	0	0	0
		0.1581D 00	3	3	2	1	1	0	2	0
		0.1936D 00	7	7	0	0	0	0	0	0
		0.5898D 00	11	11	0	0	0	0	0	0
		-.1127D 00	3	7	2	1	1	0	2	0
		0.1189D 00	11	11	2	1	1	0	2	0
TOTAL=		0.1000000000000000D 01								

#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
2	0									
		-.2255D 00	1	3	2	1	1	0	2	0
		0.1324D 00	1	3	2	2	2	0	2	0
		0.4611D 00	3	3	0	0	0	0	0	0
		0.2446D 00	3	3	0	2	0	0	0	0
		-.1761D 00	5	5	0	0	0	0	0	0
		0.3969D 00	3	3	2	1	1	0	2	0
		-.6602D 00	11	11	0	0	0	0	0	0
		-.1304D 00	3	7	2	1	1	0	2	0
		-.1900D 00	11	11	2	1	1	0	2	0
TOTAL=		0.9999999999999995D 00								

#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
3	0									
		-.5537D 00	1	1	0	0	0	0	0	0
		0.3613D 00	1	3	2	1	1	0	2	0
		-.2355D 00	1	1	0	2	0	0	0	0
		0.1258D 00	1	3	2	3	1	0	2	0
		0.4202D 00	3	3	0	0	0	0	0	0
		0.2267D 00	1	5	2	1	1	0	2	0
		-.2912D 00	3	3	0	2	0	0	0	0
		0.1076D 00	3	3	2	1	1	0	2	0
		-.1178D 00	5	5	0	2	0	0	0	0
		0.1146D 00	3	3	2	2	2	0	2	0
		-.1021D 00	7	7	0	2	0	0	0	0
		-.2919D 00	11	11	0	2	0	0	0	0
TOTAL=		0.9999999999999998D 00								

Table XIV. (Continued)

#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
1	1									
		0.8683D 00	1	3	1	0	0	0	0	1
		0.1589D 00	1	3	1	1	1	0	2	1
		0.1601D 00	1	3	2	1	1	0	2	1
		-.1111D 00	3	5	1	1	1	0	2	1
		-.1250D 00	1	7	3	1	1	0	2	1
		0.3346D 00	3	3	2	1	1	0	2	1
		-.1081D 00	3	5	3	1	1	0	2	1
		0.1263D 00	3	5	2	1	1	0	2	1
TOTAL=		0.1000000000000000D 01								

#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
2	1									
		-.1822D 00	1	3	1	0	0	0	0	1
		-.4148D 00	1	3	1	1	1	0	2	1
		0.1962D 00	1	3	1	2	2	0	2	1
		0.4583D 00	3	5	1	0	0	0	0	1
		0.1122D 00	3	5	1	1	1	0	2	1
		0.6519D 00	3	3	2	1	1	0	2	1
		0.1491D 00	3	5	3	1	1	0	2	1
		-.1093D 00	3	3	2	2	2	0	2	1
		-.1435D 00	3	7	4	2	2	0	4	1
		0.1138D 00	3	5	2	1	1	0	2	1
TOTAL=		0.1000000000000000D 01								

#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
3	1									
		-.2079D 00	1	3	1	0	0	0	0	1
		0.4029D 00	1	3	1	1	1	0	2	1
		-.4201D 00	1	3	2	1	1	0	2	1
		0.2294D 00	1	3	2	2	2	0	2	1
		-.4035D 00	3	5	1	0	0	0	0	1
		-.1554D 00	3	5	4	2	2	0	4	1
		0.5289D 00	3	3	2	1	1	0	2	1
		-.1089D 00	3	5	3	1	1	0	2	1
		-.1039D 00	3	7	4	2	2	0	4	1
		-.1941D 00	3	5	2	1	1	0	2	1
TOTAL=		0.1000000000000000D 01								

Table XIV. (Continued)

#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
1	2									
		0.2259D 00	1	3	2	0	0	0	0	2
		-.2899D 00	1	1	0	1	1	0	2	2
		0.1079D 00	1	3	2	1	1	0	2	2
		0.1294D 00	1	3	2	2	2	0	4	2
		-.5786D 00	3	3	0	1	1	0	2	2
		-.2535D 00	3	3	2	0	0	0	0	2
		-.1956D 00	5	5	0	1	1	0	2	2
		-.1154D 00	3	3	2	2	2	0	4	2
		-.1715D 00	7	7	0	1	1	0	2	2
		-.4970D 00	11	11	0	1	1	0	2	2
TOTAL=		0.1000000000000000D 01								

#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
2	2									
		0.1200D 00	1	1	0	1	1	0	2	2
		0.1310D 00	1	3	1	1	1	0	2	2
		0.2571D 00	1	3	2	1	1	0	2	2
		0.1339D 00	3	3	0	2	2	0	2	2
		-.8197D 00	3	3	2	0	0	0	0	2
		0.1339D 00	3	7	4	1	1	0	2	2
		0.2982D 00	11	11	0	1	1	0	2	2
		0.1114D 00	11	11	2	0	0	0	0	2
TOTAL=		0.1000000000000000D 01								

#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
3	2									
		0.6543D 00	1	3	2	0	0	0	0	2
		-.1358D 00	1	3	1	1	1	0	2	2
		0.1490D 00	1	3	2	1	1	0	2	2
		0.1231D 00	1	1	0	2	2	0	2	2
		0.1015D 00	3	5	4	1	1	0	2	2
		0.2750D 00	3	3	0	2	2	0	2	2
		0.2943D 00	3	3	2	0	0	0	0	2
		-.3772D 00	3	3	2	1	1	0	2	2
		0.1857D 00	11	11	0	1	1	0	2	2
		0.1979D 00	11	11	0	2	2	0	2	2
TOTAL=		0.1000000000000000D 01								

Table XIV. (Continued)

#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
4	2									
		0.5000D 00	1	3	2	0	0	0	0	2
		-.2425D 00	1	1	0	2	2	0	2	2
		-.4777D 00	3	3	0	2	2	0	2	2
		-.1373D 00	3	3	2	0	0	0	0	2
		-.1832D 00	5	5	0	2	2	0	2	2
		-.1587D 00	7	7	0	2	2	0	2	2
		0.1471D 00	11	11	0	1	1	0	2	2
		-.4916D 00	11	11	0	2	2	0	2	2
		0.1434D 00	11	11	2	1	1	0	2	2
TOTAL=		0.9999999999999999D 00								

#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
5	2									
		0.3212D 00	3	3	0	1	1	0	2	2
		-.1032D 00	3	3	2	0	0	0	0	2
		-.1478D 00	3	3	2	1	1	0	2	2
		0.1248D 00	3	3	2	2	2	0	4	2
		-.2201D 00	11	11	0	1	1	0	2	2
		-.7475D 00	11	11	2	0	0	0	0	2
		0.2182D 00	11	11	2	1	1	0	2	2
		-.2974D 00	11	11	4	1	1	0	2	2
TOTAL=		0.1000000000000000D 01								

#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
1	3									
		0.1459D 00	1	3	2	1	1	0	2	3
		0.2198D 00	1	3	2	2	2	0	2	3
		0.2232D 00	1	3	2	2	2	0	4	3
		0.1312D 00	3	3	0	3	3	0	3	3
		-.8529D 00	3	3	2	1	1	0	2	3
		0.1850D 00	3	7	3	0	0	0	0	3
TOTAL=		0.1000000000000000D 01								

Table XIV. (Continued)

#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
2	3									
		-.1301D 00	1	5	3	0	0	0	0	3
		0.2829D 00	1	3	1	1	1	0	2	3
		-.6661D 00	1	3	2	1	1	0	2	3
		-.1467D 00	1	3	2	2	2	0	2	3
		0.1178D 00	1	3	1	2	2	0	4	3
		-.2960D 00	3	3	0	3	3	0	3	3
		-.3517D 00	3	5	3	0	0	0	0	3
		-.1654D 00	3	3	2	1	1	0	2	3
		0.2105D 00	3	3	2	2	2	0	2	3
		0.2955D 00	3	3	2	2	2	0	4	3
		-.1195D 00	11	11	0	3	3	0	3	3
TOTAL=		0.1000000000000000D 01								

#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
3	3									
		-.7841D 00	1	3	1	1	1	0	2	3
		-.2433D 00	1	3	2	1	1	0	2	3
		-.1188D 00	1	3	1	2	2	0	4	3
		-.1335D 00	1	3	2	2	2	0	4	3
		0.1945D 00	1	7	3	0	0	0	0	3
		0.1167D 00	3	5	1	2	2	0	4	3
		-.1226D 00	3	3	0	3	3	0	3	3
		0.1432D 00	3	5	3	0	0	0	0	3
		-.1846D 00	3	3	2	1	1	0	2	3
		0.2805D 00	3	3	2	2	2	0	2	3
		-.1075D 00	3	3	2	2	2	0	4	3
		-.1238D 00	3	5	2	2	2	0	4	3
TOTAL=		0.1000000000000000D 01								

Table XIV. (Continued)

#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
2	4									
		0.2999D 00	1	3	2	1	1	0	2	4
		-.2633D 00	1	1	0	2	2	0	4	4
		0.1356D 00	1	3	2	2	2	0	4	4
		0.1265D 00	1	3	2	3	3	0	6	4
		0.1306D 00	3	5	4	0	0	0	0	4
		-.5304D 00	3	3	0	2	2	0	4	4
		-.3582D 00	3	3	2	1	1	0	2	4
		-.1769D 00	5	5	0	2	2	0	4	4
		-.1103D 00	3	3	2	3	3	0	6	4
		-.1539D 00	7	7	0	2	2	0	4	4
		0.1069D 00	3	7	2	1	1	0	2	4
		-.4356D 00	11	11	0	2	2	0	4	4
		-.1240D 00	11	11	2	1	1	0	2	4
TOTAL=		0.9999999999999997D 00								

#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
3	4									
		-.1018D 00	1	3	2	1	1	0	2	4
		0.2506D 00	3	3	2	1	1	0	2	4
		-.1648D 00	11	11	0	2	2	0	4	4
		-.2552D 00	11	11	6	1	1	0	2	4
		-.7984D 00	11	11	4	0	0	0	0	4
		-.3123D 00	11	11	2	1	1	0	2	4
		0.1613D 00	11	11	4	1	1	0	2	4
TOTAL=		0.1000000000000002D 01								

#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
4	4									
		0.1301D 00	1	3	2	1	1	0	2	4
		-.1752D 00	1	1	0	2	2	0	4	4
		-.1554D 00	1	3	1	2	2	0	4	4
		-.1539D 00	1	3	2	2	2	0	4	4
		-.1292D 00	3	3	0	2	2	0	4	4
		-.1980D 00	3	7	4	0	0	0	0	4
		0.7225D 00	3	3	2	1	1	0	2	4
		-.1053D 00	5	5	0	2	2	0	4	4
		-.3289D 00	11	11	0	2	2	0	4	4
		0.1110D 00	11	11	6	1	1	0	2	4
		0.3326D 00	11	11	4	0	0	0	0	4
TOTAL=		0.1000000000000011D 01								

Table XIV. (Continued)

#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
5	4									
		-.1273D 00	1	7	4	0	0	0	0	4
		0.6646D 00	1	3	2	1	1	0	2	4
		-.1121D 00	1	3	2	2	2	0	2	4
		-.1298D 00	1	3	1	2	2	0	4	4
		0.1067D 00	1	3	2	2	2	0	4	4
		0.3480D 00	3	5	4	0	0	0	0	4
		0.1634D 00	3	5	4	1	1	0	2	4
		0.1724D 00	3	3	0	2	2	0	4	4
		0.1897D 00	3	3	0	3	3	0	4	4
		0.2276D 00	3	3	2	1	1	0	2	4
		0.1581D 00	3	3	2	2	2	0	2	4
		-.2384D 00	3	3	2	2	2	0	4	4
		0.2825D 00	11	11	0	2	2	0	4	4
		0.1168D 00	11	11	0	3	3	0	4	4
		-.1969D 00	11	11	4	0	0	0	0	4
TOTAL=		0.1000000000000000D 01								

#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
6	4									
		-.3367D 00	1	11	5	1	1	0	2	4
		-.1365D 00	1	11	5	2	2	0	4	4
		-.1758D 00	1	11	6	2	2	0	4	4
		-.2390D 00	3	11	7	2	2	0	4	4
		0.2792D 00	3	11	6	1	1	0	2	4
		-.1743D 00	3	11	6	2	2	0	2	4
		-.5926D 00	3	11	5	1	1	0	2	4
		0.1208D 00	3	11	5	2	2	0	2	4
		0.3623D 00	3	11	4	1	1	0	2	4
		0.1696D 00	3	11	4	2	2	0	4	4
		-.2231D 00	5	11	4	0	0	0	0	4
		-.1791D 00	7	11	4	0	0	0	0	4
TOTAL=		0.1000000000000000D 01								

Table XIV. (Continued)

#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
3	6									
		0.2292D 00	11	11	8	1	1	0	2	6
		0.9198D 00	11	11	6	0	0	0	0	6
		0.2897D 00	11	11	4	1	1	0	2	6
TOTAL=		0.1000000000000001D 01								

#	SPIN	COEF.	J1	J2	J	N	L	NU	R	I
4	6									
		-.3622D 00	1	3	2	2	2	0	4	6
		0.2333D 00	1	1	0	3	3	0	6	6
		-.1505D 00	1	3	2	3	3	0	6	6
		-.2078D 00	3	5	4	1	1	0	2	6
		0.5218D 00	3	3	0	3	3	0	6	6
		-.1202D 00	3	7	4	1	1	0	2	6
		0.3976D 00	3	3	2	2	2	0	4	6
		0.1611D 00	5	5	0	3	3	0	6	6
		0.1194D 00	5	11	6	0	0	0	0	6
		0.1405D 00	7	7	0	3	3	0	6	6
		-.1255D 00	3	7	2	2	2	0	4	6
		0.3838D 00	11	11	0	3	3	0	6	6
		0.1339D 00	11	11	2	2	2	0	4	6
TOTAL=		0.9999999999999996D 00								

significantly (to the level of .1% which we have written out), as opposed to what we found in the N = 84 case.

Electromagnetic properties

There are even fewer experimental numbers to compare with here than for the N = 84 nuclei. In Table XV we give the theoretical $B(E2;2_1^+ \rightarrow 0_1^+)$ transition, the 2_1^+ electric quadrupole moment, and the 2_1^+ magnetic dipole moment, calculated for each nucleus. Also shown are the experimentally determined B(E2) for ^{136}Ba (82b) and the quadrupole moment for the same nucleus (82b). Agreement is seen to be quite good. It is important to note that in Reference (87) agreement with experiment is obtained by arbitrarily renormalizing $B(E2;2_1 \rightarrow 0_1)$ to the experimental value, while we do not have to resort to such a measure. We have used $e_n^{eff} = e$ here in contrast to the N = 84 case. The transitions with which we can compare are predicted to be largely collective, however the B(E2) ratio discussed below is sensitive to the value of e_n^{eff}.

We can also compare the theoretical and experimental B(E2) ratio

$$\frac{B(E2;2_2^+ \rightarrow 0_1^+)}{B(E2;2_2^+ \rightarrow 2_1^+)}$$

which we do in Table XVI. The inhibition of the crossover transition $2_2^+ \rightarrow 0_1^+$ is seen to be in reasonably good agreement with experiment, except that the model predicts too much inhibition in ^{134}Xe. This would indicate that the model state is predicted to be too much of a two-phonon state with insufficient mixing of particle states since a purely

collective two-phonon E2 transition is forbidden.

The calculated B(E2) and B(M1) values are given in Tables XVII and XVIII for these N = 80 nuclei. Throughout, $e_n^{eff} = e$ and $g_S^{eff} = g_S^{free}$.

It is interesting to note that for the form of the interaction we used (pairing residual plus quadrupole-quadrupole particle surface), the form of the matrix was such that altering the sign of the coupling ξ does not change the energy spectrum, although it changes the relative phases of the eigenfunctions from the second diagonalization. Hence the sign of ξ affects the electromagnetic properties and most dramatically the electric quadrupole properties. This result, of course, is true for both the two-particle (N = 84) nuclei as well as for the two-hole (N = 82) nuclei. Thus the electromagnetic (principally electric) transition rates dictate the correct choice of sign of ξ which is the same for both particles and holes although for the latter ξ is less in magnitude.

Table XV. Theoretical and experimental values of $B(E2;2_1^+ \rightarrow 0_1^+)$, $Q(2_1^+)$, and $\mu(2_1^+)$ for the N = 80 nuclei with a residual pairing interaction. The values used were $e_n^{eff} = 0.5\ e$, $g_S = g_S^{free}$, $g_\ell = 0$, $g_R = Z/A$

	$B(E2;2_1^+ \rightarrow 0_1^+)$		$Q(2_1^+)\,(e^2b^2)$		$\mu(2_1^+)\,(nm)$	
	Theory	Experiment	Theory	Experiment	Theory	Experiment
^{134}Xe	.0528		+.0707		.355	
^{136}Ba	.0718	.0836 ± .0022	+.164	+.34 ± .52 -.19 ± .17 +.43 ± .52	.330	
^{138}Ce	.0805		+.212		.328	
^{140}Nd	.0870		+.230		.337	

Table XVI. Theoretical and experimental values for the ratio $B(E2;2_2^+ \rightarrow 0_1^+)/B(E2;2_2^+ \rightarrow 2_1^+)$ in the N = 80 nuclei. The residual interaction is a pairing interaction and the electromagnetic parameters are as given for Table XV

	Theory	Experiment
^{134}Xe	~ 0	.026
^{136}Ba	.038	.023
^{138}Ce	.0270	.043
^{140}Nd	.0228	-

Table XVII. Electric quadrupole transitions for the N = 80 nuclei with a residual pairing interaction. The values of $B(E2;i \rightarrow f)$ and $\langle f||M(E2)||i\rangle$ were calculated using $e_n^{eff} = e$. The upper number of each pair of numbers is the B(E2) value

i	f	$^{134}_{Xe}$	$^{136}_{Ba}$	$^{138}_{Ce}$	$^{140}_{Nd}$
2_1	2_1	.0017 +.0417	.0094 +.0970	.0157 +.1253	.0184 +.1357
	0_1	.0528 .5136	.0718 -.5991	.0805 .6345	.0870 .6698
2_2	0_1	.0000 -.0080	.0007 +.0589	.0012 +.0775	.0014 .0842
	2_1	.0182 -.1348	.0184 .1355	.0445 -.2109	.0641 -.2532
0_2	2_1	.0497 -.0996	.0468 -.0967	.0596 .1091	.0669 .1156
	2_2	.0402 +.0897	.0613 -.1107	.0773 -.1243	.0842 -.1298
2_3	2_1	.0625 +.2501	.0510 .2257	.0416 -.2039	.0329 .1814
	2_2	.0045 -.0668	.0105 +.1026	.0041 +.0644	.0015 -.0387
	0_1	.0000 .0075	.0001 +.0196	.0001 +.0172	.0000 -.0097
4_1	2_1	.1027 +.4300	.1358 -.4943	.1490 +.5179	.1598 +.5363
	2_2	.0001 -.0156	.0003 .0229	.0005 -.0284	.0005 .0307

Table XVII. (Continued)

i	f	$^{134}_{Xe}$	$^{136}_{Ba}$	$^{138}_{Ce}$	$^{140}_{Nd}$
4_2	4_1	.0068 .0825	.0056 -.0749	.0108 -.1038	.0145 -.1204
	2_1	.0000 .0022	.0000 .0034	.0001 .0159	.0003 .0213
	2_2	.0293 -.2296	.0162 -.1706	.0364 +.2560	.0495 .2984
	2_3	.0009 .0408	.0040 -.0850	.0018 +.0564	.0003 -.0218

Table XVIII. Magnetic dipole transitions for the N = 80 nuclei with a residual pairing interaction. The values of $B(M1; i \to f)$ and $\langle f||M(M1)||i\rangle$ are computed for $g_S = g_S^{free}$, $g_\ell = 0$, $g_R = Z/A$. The upper number of each pair of numbers is the B(M1) value.

i	f	^{134}Xe	^{136}Ba	^{138}Ce	^{140}Nd
2_1	2_1	.2125 .4610	.1973 .4441	.1960 .4426	.2017 .4491
2_2	2_1	-.0020 .0445	.0054 +.0737	.0001 -.0305	.0000 -.0014
2_3	2_2	.0429 +.2072	.0334 -.1828	.1020 -.3194	.1273 +.3574
	2_1	.0096 -.0977	.0348 -.1865	.0418 +.2045	.0415 -.2038
2_4	2_1	.0117 .1080	.0136 -.1163	.0183 .1353	.0208 .1421
	2_2	.0047 -.0685	.0000 -.0008	.0056 -.0735	.0149 -.1225
	2_3	.1350 .3675	.1203 -.3469	.1183 -.3440	.1110 +.3332
4_2	4_1	.0081 -.0899	.0130 -.1141	.0075 -.0863	.0052 -.0722

CHAPTER 4. SUMMARY AND CONCLUSIONS

The primary objective of this thesis has been to investigate the contribution of four phonon excitation states of the core in the 2-particle plus vibrating core unified model, and to develop a more realistic and vigorous basis truncation procedure to use with this model. As a secondary objective, we considered the usefulness of a so-called "realistic" residual interaction which consisted of "bare" matrix elements (first order in a perturbation expansion), as opposed to an extremely simple pairing interaction.

To carry out this plan, we first had to construct an explicit form for the multiplicity resolved U(5) eigenfunctions of the core in a body fixed reference frame. The results of that investigation are contained in Reference (60), and have served as the foundation for both this thesis, and the dissertation work of one of our co-authors in Reference (60), F. Margetan.

These eigenfunctions were used to construct matrix elements of the collective surface coordinate α between arbitrary (symmetric) U(5) eigenfunctions, and to the best of our knowledge this is the first such solution.

A basis truncation procedure to treat the core and coupled single particle states on an equal footing was devised. This consisted of a two-step diagonalization of the unified model Hamiltonian.

As in most research in nuclear theory these days, the computer played a large part, both in enabling the calculation to be carried out,

and in imposing limits on what could be done. A large part of the research we did consisted of writing a large code to construct the basis states, generate matrix elements of the Hamiltonian between these states, diagonalize these matrices for varying values of the models parameters $\hbar\omega$ and ξ until agreement with experimental spectra was found, and finally calculate the electromagnetic properties predicted by this model. Since the model requires repeated diagonalization of large matrices, it is quite expensive to use to fit nuclei, which is probably why previous investigators have been content to explain many nuclei with one set of parameters. We found that the electromagnetic properties of similar looking spectra could vary considerably, so using one set of parameters for many nuclei cannot be expected to yield ideal results.

When we applied this model to N = 84 nuclei with a pairing interaction, we found that both the low-lying positive parity ($\leq$ 4 MeV) and electromagnetic properties could be fitted very well, and that the contribution of four phonon core states to some levels was significant.

For these same nuclei, the realistic matrix elements did a very poor job for both the spectra and electromagnetic properties. The mixing induced by these matrix elements is simply too small; they do not contain enough pairing-type interaction. The effect of higher order diagrams in the G-matrix expansion would seem to be needed and would be interesting to investigate in this manner.

The low-lying experimental spectra for the N = 80 nuclei were reproduced closely. The few electromagnetic properties seem to be explained reasonably well. This is an area where more experimental

electromagnetic data would be very useful. The four phonon states did not seem nearly as important as we found them to be in the N = 84 nuclei, and when some more B(E2)'s and such are reported experimentally it would be worthwhile to redo some of these fits in a basis restricted to three phonon states. There remain quite a few regions where this model could be employed as it stands. For example, there one could consider nuclei with N = 30 (two neutrons) and N = 26 (two neutron holes), or N = 52 and 48. It would be interesting to extend the treatment of the core to include anharmonic terms and to extend the particle basis to include more than two particles outside the core. Both of these procedures would increase the basis size rapidly and considerably greater computer power than presently available at Iowa State University would be required.

Since the electromagnetic properties of this (or any) model are more sensitive to the parameters (as reflected in the eigenfunctions) than the energy levels, it would be interesting (but very expensive in computer time) to fit both the levels and say the electric quadrupole and magnetic dipole moments simultaneously. Our point here is that discrepancies in the electromagnetic properties for a given fit do not necessarily imply a failing in the model, as another almost equally good level fit (better agreement for some levels and worse for others but with a similar chi-squared value) would in some cases produce quite different electromagnetic properties.

In conclusion, the unified model with a simple pairing residual interaction for two extra-core particles has been found to account for most of the reported experimental properties of the N = 84 and N = 80

nuclei. Although the four phonon contribution is not always needed, it was found to have a non-negligible contribution in some cases, and a means for including these contributions and still maintaining a manageable-sized basis was demonstrated.

BIBLIOGRAPHY

1. J. Chadwick, Proc. Royal Soc. Lon. A136, 692 (1932).

2. M. E. Rose, Multipole Moments (Wiley, New York, 1954).

3. J. Thibaud, Compt. Rend. 191, 656 (1930).

4. E. Teller and J. A. Wheeler, Phys. Rev. 53, 778 (1938).

5. N. Bjerrum, Nernst Festchrift (Knapp, Halle, 1912).

6. J. W. Mihelich, G. Scharff-Goldhaber, and M. McKeown, Phys. Rev. 94, 794 (1954).

7. T. Schmidt, Z. Physik 106, 358 (1937).

8. E. Feenburg, Phys. Rev. 75, 320 (1949).

9. O. Haxel, J. D. Jensen, and H. E. Seuss, Phys. Rev. 75, 1776 (1949).

10. M. G. Mayer, Phys. Rev. 74, 235 (1948).

11. M. G. Mayer, Phys. Rev. 75, 1969 (1949).

12. M. G. Mayer and J. H. D. Jensen, Elementary Theory of Nuclear Shell Structure (Wiley, New York, 1955).

13. E. Feenburg, Phys. Rev. 55, 504 (1939).

14. N. Bohr and J. A. Wheeler, Phys. Rev. 56, 426 (1939).

15. J. Rainwater, Phys. Rev. 79, 432 (1950).

16. M. Goldhaber and A. W. Sunyar, Phys. Rev. 83, 906 (1951).

17. A. Bohr and B. R. Mottelson, Phys. Rev. 89, 316 (1953).

18. L. D. Landau, J. Phys. (USSR) 5, 71 (1941).

19. A. Bohr and B. R. Mottelson, Nuclear Structure, vol. II, (Benjamin, Reading, Mass., 1975).

20. A. Bohr, Kgl. Danske Videnskab Selskab Mat. Fys. Medd. 26, no. 14 (1952).

21. A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab Selskab Mat. Fys. Medd. 27, no. 16 (1953).

22. A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab Selskab Mat. Fys. Medd. 28, no. 1 (1954).

23. D. C. Choudhury, Kgl. Sanske Videnskab Selskab Mat. Fys. Medd. 28, no. 4 (1954).

24. K. W. Ford and C. Levinson, Phys. Rev. 100, 1 (1955).

25. B. J. Raz, Phys. Rev. 114, 1116 (1958).

26. E. Fermi, Nuclear Physics (University of Chicago Press, Chicago, 1974).

27. G. Alga, Struttura dei Nuclei e Reasini Nucleari - Proceedings of the International School of Physics, Enrico Fermi. Corso XL, edited by M. Jean (Academic Press, New York, 1969).

28. G. Ialongo, thesis, New York University (1966).

29. G. Scharff-Goldhaber and J. Weneser, Phys. Rev. 98, 212 (1955).

30. A. Covello and G. Sartoris, private communication, cited in Struttura dei Nuclei e Reasini Nucleari - Proceedings of the International School of Physics, Enrico Fermi. Corso XL, edited by M. Jean (Academic Press, New York, 1969).

31. D. C. Choudhury and T. F. O'Dwyer, Nucl. Phys. A93, 300 (1967).

32. K. Heyde and P. J. Brussaard, Nucl. Phys. A104, 81 (1967).

33. K. Heyde and G. Vanden Berghe, Nucl. Phys. A126, 381 (1968).

34. K. Heyde and P. J. Brussaard, Z. Physik 259, 15 (1973).

35. G. Vanden Berghe, K. Heyde, and M. Waroquier, Nucl. Phys. A165, 662 (1971).

36. G. Vanden Berghe and K. Heyde, Nucl. Phys. A163, 478 (1971).

37. M. Conjeaud, S. Harar, and Y. Cassagnou, Nucl. Phys. A117, 449 (1968).

38. T. Paradellis and S. Hontzeas, Nucl. Phys. A140, 400 (1970).

39. I. V. Goldstein and A. G. dePinho, Can. J. Phys. 49, 1794 (1971).

40. H. J. Mang, F. Krmpotic, and S. M. Abecasis, Z. Physik 262, 39 (1973).

41. V. Lopac, Nucl. Phys. A155, 515 (1970).

42. E. Degrieck and G. Vanden Berghe, Nucl. Phys. A231, 141 (1974).

43. G. Alga, F. Krmpotic, and V. Lopac, preprint IRB-TP-2-67 (1967).

44. E. Degrieck and G. Vanden Berghe, Z. Physik 265, 393 (1973).

45. G. Alga, V. Paar, and V. Lopac, Phys. Lett. 43B, 459 (1973).

46. G. Vanden Berghe, Z. Physik A272, 245 (1975).

47. L. J. Alquist, W. C. Schick, W. L. Talbert, and S. A. Williams, Phys. Rev. C13, 1277 (1976).

48. V. Paar, Nucl. Phys. A147, 369 (1970).

49. K. Heyde, M. Waroquier, and H. Vincx, Nucl. Phys. A234, 216 (1974).

50. G. Vanden Berghe, Z. Physik A274, 51 (1975).

51. G. Do Dang, G. J. Dreiss, R. M. Dreizler, A. Klein, and Chi-Shiang Wu, Nucl. Phys. A114, 481 (1968).

52. B. Castel, K. W. C. Stewart, and M. Harvey, Can. J. Phys. 48, 1490 (1970).

53. G. Gneuss, U. Mosel, and W. Greiner, Phys. Lett. 30B, 397 (1969).

54. G. Gneuss, W. Greiner, and U. Mosel, Phys. Lett. 32B, 161 (1970).

55. G. Gneuss and W. Greiner, Nucl. Phys. A171, 449 (1971).

56. F. Margetan, thesis, Iowa State University (1977).

57. P. O. Lipas and J. Savolainen, Nucl. Phys. A130, 77 (1969).

58. P. Haapakoski, T. Honkaranta, P. O. Lipas, Phys. Lett. 31B, 493 (1970).

59. P. O. Lipas, P. Haapakoski, and T. Honkaranta, Physica Scripta 13, 339 (1976).

60. T. M. Corrigan, F. J. Margetan, and S. A. Williams, Phys. Rev. C14, 2279 (1976).

61. J. M. Eisenberg and W. Greiner, Nuclear Models (American Elsevier, New York, 1970).

62. H. Enge, Introduction to Nuclear Physics (Addison-Wesley, Reading, Mass., 1966).

63. B. Mottelson, Rev. Mod. Phys. 48, 375 (1976).

64. A. de-Shalit and H. Feshbach, Theoretical Nuclear Physics, Volume I: Nuclear Structure (John Wiley and Sons, New York, 1974).

65. J. P. Davidson, Collective Models of the Nucleus (Academic Press, New York, 1968).

66. N. Kemmer, D. L. Pursey, and S. A. Williams, J. Math. Phys. 9, 1224 (1968).

67. D. L. Pursey and S. A. Williams, J. Math. Phys. 9, 1230 (1968).

68. A. de-Shalit and I. Talmi, Nuclear Shell Theory (Academic Press, New York, 1963).

69. L. S. Kisslinger and R. A. Sorensen, Rev. Mod. Phys. 35, 853 (1963).

70. A. L. Goodman, J. P. Vary, and R. A. Sorensen, Phys. Rev. C13, 1674 (1976).

71. M. E. Rose, Elementary Theory of Angular Momentum (John Wiley and Sons, New York, 1957).

72. B. G. Wybourne, Classical Groups for Physicists (John Wiley and Sons, New York, 1957).

73. E. D. Rainville, Special Functions (Macmillan, New York, 1963).

74. H. W. Taylor, B. Singh, F. S. Prato, and R. McPherson, Nucl. Data A9, 1 (1971).

75. G. J. Bassinger, W. C. Schick, and W. L. Talbert, Phys. Rev. C11, 1755 (1975).

76. M. Behar, Z. W. Grabowski, and S. Raman, Nucl. Phys. A219, 516 (1974).

77. S. Antman, H. Petterson, A. Aehleu, and I. Adam, Z. Physik 237, 292 (1970).

78. W. W. Bowman, D. R. Haenni, and T. T. Sugihara, Phys. Rev. C7, 1686 (1973).

79. Nucl. Data Sheets 15, 203 (1975).

80. J. M. Gualda, R. N. Saxena, F. C. Zawislak, Nucl. Phys. A234, 357 (1974).

81. Nucl. Data Sheets 13, 191 (1974).

82a. R. A. Meyer and R. D. Griffioen, Phys. Rev. 186, 1220 (1969).

82b. J. R. Kearns and J. X. Saladin, Phys. Rev. C6, 1016 (1972).

83. Nucl. Data Sheets 18, 167 (1976).

84. G. M. Julian and T. E. Fessler, Phys. Rev. C3, 751 (1971).

85. M. R. Zalutsky and E. S. Macias, Phys. Rev. C12, 997 (1975).

86. Nucl. Data Sheets 12, 343 (1974).

87. M. Meyer-Levy and V. Lopac, Phys. Rev. C8, 829 (1973).

88. D. M. Brink and G. R. Satchler, Angular Momentum (Oxford University Press, London, 1968).

89. E. K. Warburton, D. E. Alburger, D. H. Wilkinson, and J. M. Spear, Phys. Rev. 129, 2191 (1963).

90. S. Gartenhous and C. Schwartz, Phys. Rev. 108, 482 (1957).

91. S. Falieros and R. Ferrel, Phys. Rev. 116, 660 (1959)

92. B. R. Barrett and M. W. Kirson in Advances in Nuclear Physics, edited by M. Baranger and Eric Bogt (Plenum, New York, 1973).

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APPENDIX A

We list some of the conventions we use for R(3) tensor operators, wavefunctions, and their matrix elements.

First, a function is said to transform contragradiently under the contravariant Irreducible Representation (IR) $[\lambda]$ of R(3) it satisfies

$$T_{\lambda\mu} = \sum_{\nu} D^{\lambda}_{\mu\nu} T'_{\lambda\nu} \qquad \text{A.1}$$

and to transform cogradiently under the covariant $[\lambda]^*$ IR of R(3) it it satisfies

$$T_{\lambda}^{\mu} = \sum_{\nu} D^{\lambda *}_{\mu\nu} T_{\lambda}^{'\nu} \qquad \text{A.2}$$

where the upper or lower index distinguishes the two cases.

The transformation which reduces the reducible representation $[\lambda_1] \times [\lambda_2]$ into its irreducible representations (i.e., couples angular momenta) are the usual Clebsch-Gordan coefficients satisfying

$$|JM\rangle = \sum_{m_1 m_2} c(J_1 J_2 J; m_1 m_2 M) |j_1 m_1\rangle |j_2 m_2\rangle \qquad \text{A.3}$$

which also satisfy the orthogonality relations

$$\begin{aligned} \sum_{m_1 m_2} c(j_1 j_2 j; m_1 m_2 m)\, c(j_1 j_2 j'; m_1 m_2 m) &= \delta_{j\,j'} \\ \sum_{J} c(j_1 j_2 j; m_1 m_2 m)\, c(j_1 j_2 j; m_1' m_2' m) &= \delta_{m_1 m_1'} \delta_{m_2 m_2'} \end{aligned} \qquad \text{A.4}$$

More convenient in the sense of easier-to-remember symmetry relations

are the 3-j symbols of Wigner, where

$$c(j_1 j_2 j; m_1 m_2 m) = (-1)^{j_1 j_2} \sqrt{2j+1} \begin{pmatrix} j_1 & j_2 & j \\ m_1 & m_2 & -m \end{pmatrix} \qquad \text{A.5}$$

The calculation of matrix elements of the components of a tensor operator is greatly simplified by the well-known Wigner-Eckart (71) theorem

$$\langle J'M'|T_{LK}|JM\rangle = c(JLJ';MKM')\langle J'||T_L||J\rangle \quad . \qquad \text{A.6}$$

This deserves a few comments. First, the form we use is not universal by any means, many authors (64) prefer to define the reduced matrix elements in terms of 3-j symbols, i.e.,

$$\langle J'M'|T_{LK}|JM\rangle = (-1)^{j'-m'} \begin{pmatrix} j' & L & j \\ -m' & K & m \end{pmatrix} \overline{\langle J'||T_L||J\rangle}$$

the two forms of reduced matrix elements are then related by

$$\langle J'||T_L||J\rangle = \frac{1}{\sqrt{2J'+1}} \overline{\langle J'||T_L||J\rangle}$$

Second, the reduced matrix elements are defined in terms of the contravariant components of the tensor, as we emphasized in the derivation of matrix elements of α_μ.

Finally, the Wigner-Eckart theorem is a more general result than is implied by our use here for R(3). For any compact (or indeed any finite) group G with irreducible representations $[\Lambda_i]$ with weights λ_i (like J and M in R(3)), we may write (72)

$$\langle \Lambda_1 \lambda_1 | T(\Lambda\lambda) | \Lambda_2 \lambda_2 \rangle = \sum_a \langle a\Lambda_1 \lambda_1 | \Lambda\lambda_1 \Lambda_2 \lambda_2 \rangle \langle a\Lambda_1 || T(\Lambda) || \Lambda_2 \rangle \qquad \text{A.7}$$

where a is any other label on the states. Furthermore, we may apply the W-E theorem to nested chains of subgroups ($G_1 \supset G_2 \supset \ldots \supset G_n$) by applying Rachas factorization lemma (3) which reduces the coupling coefficients of the larger group to those of its subgroup, and another factor called an isoscalar factor (3). The calculation of matrix elements then reduces to the evaluation of three distinct parts: (1) the reduced matrix elements of the smallest group in the chain, (2) the coupling coefficients for this group, and (3) the isoscalar factors for the chain. We could formally look at our calculation this way. Then noting that α is a mixed tensor in the subgroup chain $U(5) \supset SU(5) \supset R(5) \supset R(3)$, because of Equation 2.65 and the fact that creation operator has tensor properties in this chain of (60)

$$b^+ \begin{matrix} [10000] \\ [1000] \\ [10] \\ [2] \end{matrix}$$

we would write

$$\langle N' \ell' \nu' R' M' | b^+_{\mu} | N \ell \nu R M \rangle$$

$$= \langle N' \ell' \nu' R' | 1102, N\ell\nu R \rangle \langle R' || b^+ || R \rangle c(R' 2 R' M' \mu M) \quad .$$

Then, from Equation 2.64 we would identify

$$\langle R' || b^+ || R \rangle = (2R' + 1)^{-1}$$

and $\langle N' \ell' \nu' R' | 1102; N\ell\nu R \rangle$ = every thing else in Equation 2.64. (Although to really follow this procedure we would have to factor this into isoscalar factors.)

Getting back to R(3), we write the matrix element of a coupled tensor operator in a coupled space

$$T_{LK} = \sum_{m_1 m_2} c(L_1 L_2 L; M_1 M_2 K) T_{L_1 M_1}(1) T_{L_2 M_2}(2) \quad , \qquad \text{A.8}$$

where T(1) and T(2) act on different spaces, as

$$\langle (j_1' j_2') J' M' || T_{LK} || (j_1 j_2) J \rangle$$

$$= \hat{L}\, \hat{J}\, \hat{J}_1'\, \hat{J}_2' \begin{Bmatrix} J_2 & J_1 & J \\ J_2' & J_1' & J' \\ L_2 & L_1 & L \end{Bmatrix} \langle J_1' || T_{L_1}(1) || J_1 \rangle \langle J_2' || T_{L_2}(2) || J_2 \rangle \quad . \qquad \text{A.9}$$

In Equation A.9 { } is the well-known 9-j symbol (88) which is the unitary transformation relating possible couplings of 4 angular momenta.

The special cases of Equation A.9 which we use are for a scalar operator (L = 0) and an operator acting on only one space ($T_{LK}(1)$). For these we have

$$\langle (j_1' j_2') J' || T_{00} || (j_1 j_2) J \rangle$$

$$= \langle (j_1' j_2') J' || \sum_{\mu} T_L^{\mu}(1) T_{L\mu}(2) || (j_1 j_2) J \rangle$$

$$= \delta_{J'J} (1)^{j_1+j_2'+J} \left[(2j_1'+1)(2j_2'+1)\right]^{\frac{1}{2}} \begin{Bmatrix} j_1 & j_1' & L \\ j_2' & j_2 & J \end{Bmatrix}$$

$$\times \langle j_1' || T_L(1) || j_1 \rangle \langle j_2' || T_L(2) || j_2 \rangle \qquad \text{A.10}$$

and

$$\langle (j_1' j_2')J' || T_L(1) || (j_1 j_2)J \rangle$$

$$= (-1)^{j_2+L+j_1'+J} \left[(2j_1'+1)(2J+1)\right]^{\frac{1}{2}} \begin{Bmatrix} j_1 & j_1' & j_2 \\ J' & J & L \end{Bmatrix}$$

$$\langle j_1' || T_L(1) || j_1 \rangle \delta_{j_1 j_2} \qquad \text{A.11}$$

where the 6-j coefficient is related to the 9-j and 3-j by

$$\begin{Bmatrix} j_1 & j_2 & J \\ j_1' & j_2' & J \\ K & K & 0 \end{Bmatrix} = (-1)^{j_2+j_1'+K+J} \begin{Bmatrix} j_1 & j_2 & J \\ j_2' & j_1' & K \end{Bmatrix} \left[(2J+1)(2K+1)\right]^{-\frac{1}{2}} \qquad \text{A.12}$$

and

$$\begin{Bmatrix} j_1 & j_2 & j_3 \\ \ell_1 & \ell_2 & \ell_3 \end{Bmatrix} = \Sigma (-1)^{\ell_3-j_3-j_1-j_2-\ell_1-\ell_2-m_1-m_1'} \begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & -m_3 \end{pmatrix}$$

$$\times \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1' & m_2' & m_3' \end{pmatrix} \begin{pmatrix} j_2 & \ell_1 & \ell_3 \\ m_2 & m_1' & -m_3' \end{pmatrix} \begin{pmatrix} \ell_2 & j_1 & \ell_3 \\ m_2' & m_1 & m_3' \end{pmatrix} \qquad \text{A.13}$$

Relations among these quantities are found almost everywhere; one useful place is Brink and Satchler (88).

Finally, we note that we use the form and conventions for the rotation matrices $D^{J}_{MK}(\theta_1, \theta_2, \theta_3)$ that Rose (71) does.

APPENDIX B. EFFECTIVE CHARGE

The use of effective charge for a valence nucleon is a technique designed to correct the electric multipole operator, at least in part, for the omission from the problem of part of the full particle space. The net result is to assign to the valence nucleons charges in excess of their free space values ($e_p = +1$, $e_n = 0$). Often this is done empirically. For example in the 1-p shell the value $e_p = 1.5$ and $e_n = 0.5$ have been used (89). Of course values can also be derived, with some justification, by arguments which depend upon the particular model under consideration.

In the shell model the core is inert, but when a valence nucleon undergoes a transition, the core must recoil to conserve momentum and so can induce a transition. This is the idea behind the nuclear recoil effect (90). As shown in de-Shalit and Feshbach (64), for E1 transitions the core's induced transitions can be accounted for by giving an effective charge to valence neutrons and protons,

$$\text{E1:} \qquad e_n^{\text{eff}} = +\frac{Z}{A}e$$

$$e_p^{\text{eff}} = \frac{N}{A}e$$

However for higher multipoles this effect decreases rapidly; for quadrupole transitions

$$\text{E2:} \qquad e_n^{\text{eff}} = \frac{Z}{A^2}e$$

$$e_p^{eff} = \left(1 - \frac{2}{A} + \frac{Z}{A^2}\right) e$$

This will account for much, i.e., in ^{140}Ba we have

$$e_n^{eff} = \frac{56}{(A0)^2} e = .008\ e$$

instead of the .4 e observed in this calculation.

Actually the concept is even more questionable than this for the single particle shell model. It was shown in a paper by Falieros and Ferrel (91) that for harmonic oscillator wavefunctions the quadrupole operator has vanishing matrix elements for the ground state. This is certainly at odds with experience.

Another argument for effective charge is that since the calculation is performed in a truncated Hilbert space, the effective charge accounts for contributions to the true wavefunction by components outside of the truncated space. This could apply to any model, since in all we are forced to deal with a truncated space. But, only with a real theory of effective operators as in a realistic calculation could such effects be calculated even in principle (92). If they are just treated as parameters the concept becomes questionable. This is because the effective charges are vigorously both multipole and state dependent (64), and we don't want to introduce a different effective charge for each transition!

One attempt to investigate the effect of valence space size on the effective charge in a shell model calculation (64) shows that a larger model space does indeed imply smaller effective charge.

Finally, the concept has been explained by Bohr and Mottelson (19) in terms of polarization of the core (similar to polarization of a dielectric). If we are dealing with a spherical shell model the extra nucleons cause distortion of the field. The eccentricity of the density distribution of the non-spherical field generated by the extra particles is of the order A^{-1}, so each proton acquires an extra quadrupole moment $A^{-1}Q_{sp}$. Thus, for the nucleus as a whole

$$Q_{pol} \sim \frac{Z}{A} Q_{sp} \quad .$$

Because the nuclear potential is an isovector (not isospin independent), the fact that a neutron outside of a closed shell acts more strongly on the protons within the shell than does a proton, and so acquires a greater polarization charge.

The effective operator approach is also used for g_s, the spin gyromagnetic ratio of the valence particle.